X. Bakerian Lecture.—*On the Mechanical Equivalent of Heat.*

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[Plates 3–8.]

PART I.

On the Method, Appliances and Limits of Error in the direct Determination of the Work expended in raising the Temperature of Ice-cold Water to that of Water boiling under a pressure of 29·899 inches of Ice-cold Mercury in Manchester.—By Osborne Reynolds.

The Standard of Temperature for the Mechanical Equivalent.

1. The determination by Joule, in 1849, of the expenditure of mechanical effect (772·69 lbs. falling 1 foot) necessary to raise the temperature of 1 lb. of water, weighed *in vacuo*, 1° Fahr. between the temperatures of 50° and 60° Fahr. (at Manchester), together with the second, in 1878, 772·55 ft.-lbs., to raise the temperature of 1 lb. (weighed *in vacuo*) from 60° to 61° Fahr., at the latitude of Greenwich, established once for all the existence of a physically constant ratio between the work expended in producing heat and the heat produced; while the extreme simplicity of his methods, his marvellous skill as an experimenter, and the complete system of checks he adopted, have led to the universal acceptance of the numbers he obtained as being within the limits he himself assigned (1 foot), of the true ratio of work expended in his experiments in producing heat and the heat produced as measured on the scale of the thermometer on which he spent so much time and care.

The acceptance of $J = 772$, as the mechanical equivalent of heat, amounts to the acceptance of the scale between 50 and 60 on Joule's thermometer b as the standard of temperature over this range.

Joule's thermometers are now in the custody of the Manchester Literary and Philosophical Society (having been confided to its care by Mr. A. Joule); so that

this material standard is available. But the standard of temperature actually established by JOULE is universally available wherever the British standard of length is available, together with pure water and the necessary means and skill of expending a definite quantity of work in raising the temperature of water between 50° and 60° Fahr., since in this way the scale on any thermometer may be compared with that on JOULE's.

The difficulty of access to JOULE's thermometer, and the inherent difficulty of making an accurate determination of the equivalent, have limited the number of such comparisons.

The most serious attempts have been made with the very desirable object of determining the mechanical equivalent of a thermal unit, measured on the scale of pressures of gas at constant volumes, first recognised by JOULE as the nearest approximation to absolute temperature.

The results of these comparisons have been various, all having apparently shown that JOULE's standard degree of temperature is less than the one-hundred-and-eightieth part between freezing and boiling points on the scale of pressure of gas at constant volume, the differences being from 0·1 to 1·0 per cent. JOULE himself contemplated comparing his thermometer with the scale of air pressures, but did not do so. So that only indirect comparisons have been possible.

HIRN, who was the first to follow JOULE, in one of his researches introduced a method of measuring the work done which afforded much greater facility for applying the work to the water than the falling weights used by JOULE in his first determination, and this was adopted by JOULE in his second determination. But notwithstanding the greater facilities enjoyed by subsequent observers, owing to the progress of physical appliances, the inherent difficulties remained. The losses from radiation and conduction could only be minimised by restricting the range of temperature, and this insured thermometric difficulties, particularly with the air thermometer, which, it seems, does not admit of very close reading. This, together with certain criticisms, of which some of the methods employed admit, appear to have left it still an open question what exact rise in the temperature in the scale of air pressures corresponds to the 772 ft.-lbs.

2. The research, to the method and appliances for which this paper relates, has been the result of the occurrence of circumstances which offered an opportunity, such as might not again occur, of obtaining the measure, in mechanical units, of the heat in water between the two physically fixed points of temperature to which all thermometrical measurements are referred, and of thus placing the heat as defined in mechanical units, on the same footing as the unit of heat as defined by temperature, without the intervention of scales, the intervals of which depend on the relative expansions of different materials such as mercury and glass.

It has been, so far as I am concerned, undertaken with considerable hesitation, on account of the responsibility even in attempting such a determination, and the harm

to science that might follow from further confusion owing to error in what, in spite of opportunities, must be the extremely difficult task of making such complex determinations within less than the thousandth part. These considerations, together with my inability to find the large amount of time necessary for making the observations, prevented any attempt until July, 1894. At that time Mr. W. H. MOORBY offered to devote his time to the research, and so relieve me of all responsibility except that which attached to the method and the appliances; and having, from experience, the highest opinion of Mr. MOORBY's qualifications for carrying out the very arduous research, there seemed to be no further excuse for delay, particularly as after seeing the appliances in the laboratory both Lord KELVIN and Dr. SCHUSTER expressed strongly their opinions as to the value of the research.

The Opportunity for the Research.

3. This consisted in the inclusion in the original equipment, in 1888, of the laboratory of the following appliances:—

(1.) A set of special vertical triple-expansion steam-engines, with separate boiler, closed stoke-hold, and forced blast; these engines being specially arranged to give ready access to the shafts (3 feet) above the floor, and being capable of running at any speeds up to 400 revolutions per minute, and working up to 100 H.P. (Plate 3.)

(2.) Three special hydraulic brake dynamometers, on separate shafts, between and in line with the engine shafts, with faced couplings, so that one brake shaft could be coupled with the shaft of each engine, leaving each engine to work its own shaft; or the brakes on the high-pressure and intermediate engines could be removed, and their shafts coupled by means of intermediate shafts, so that all three engines worked on the brake connected with the low-pressure engine. These brakes, which are shown (Plate 3), are separately capable of absorbing any power up to a maximum of 30 horse-power at 100 revolutions, and increasing as the cube of the speed; so that a single brake is capable of absorbing the whole power of the engine at any speed above 100 revolutions a minute.

The whole of the work is absorbed by the agitation of the water contained in the brake, while the heat so generated is discharged by a stream of water through the brake, with no other functions than of affording the means of regulating, independently, the temperature of the brake and the quantity of water in the brake. The moment of resistance of the brake at any speed is a definite function of the quantity of water in the brake. And as, except for this moment, the unloaded brake is balanced on the shaft, the load being suspended from a lever on the brake at 4 feet from the axis of the shaft, if the moment of resistance of the brake exceeds the moment of the load, the lever rises, and *vice versâ*. By making the lever actuate the valve which regulates the discharge from the brake, and thus regulate the effluent stream, the quantity of water in the brake is continually regulated to that which is just sufficient to suspend the

load with the lever horizontal, and a constant moment of resistance maintained whatever may be the speed of the engines.

(3.) Manchester town's water, of a purity expressed by not more than 3 grams of salts in a gallon, brought into the laboratory in a 4-inch main at town's pressure (50 to 100 feet), and distributed either direct from the main or at constant pressure from a service tank 10 feet above the floor of the laboratory.

(4.) Two tanks, each capable of holding 60 tons of water, one in the tower, 116 feet above the floor, the other 15 feet below the floor, connected by 4-inch rising and falling mains, each 500 feet long, passing in a chase under the floor. The rising main including a special quadruple centrifugal pump, 2 feet above the floor, capable of raising a ton a minute from the lower to the upper tank. (Shown in Plate 7.) Also a set of mercury balances, showing continually the levels of water in the two tanks, and the pressures in the rising, falling, and towns mains. (Shown in Plate 4.)

(5.) A special quadruple vortex turbine, supplied from the falling main and discharging into the lower tank, capable of exerting 1 h.p., and available for steady speed at all parts of the laboratory. (Shown in Plate 7.)

(6.) A supply of power to the laboratory by an engine and boiler, quite distinct from the experimental engine, and distributed by convenient shafting which is always running. (Shown in Plate 3.)

The Measurement of the Work.

4. Of the appliances mentioned, the brake on the low-pressure engine is the centre of interest, as it was by this that the work was measured, as well as converted into heat.

The existence of the appliances was largely due to the interest in educational work taken by Mr. WILLIAM MATHER, who, together with the other members of the firm of MATHER and PLATT, not only placed at my disposal the facilities of their works, but inspired the enthusiasm which alone rendered the execution of such novel and special work possible.

The development of the brake dynamometer, from its introduction by PRONY, has an interesting and important history, but into this it is not necessary to enter. The purpose of these dynamometers is to afford continuous frictional resistance adapted to the power exerted by the prime mover in causing a shaft to revolve, and of a kind that is definitely measurable. To fulfil the first of these conditions, the mean moment of resistance of the brake must just balance the mean moment of effort of the engine, and the means of escape of heat from the brake must be sufficient to allow all the heat generated to depart without accumulating to an extent which may interfere with the action of the appliances. In the first brakes the resistance was obtained by the friction of blocks or straps pressed against a cylindrical wheel on the shaft, and, small powers being used, radiation and air-currents round the brake were

found sufficient to carry off the heat, but, when larger powers were used, these sources of escape failed to keep the temperatures down to practical limits, which necessitated the application of currents of water to carry off the heat.

The measurement of the work was invariably accomplished by attaching the brake blocks, or straps, to a lever, or arm, so that the whole brake would be free to revolve with the brake-wheel, except for the moment of the weight of the parts which, adjusted to the power of the engine, was kept in balance by the adjustment of the pressure of the blocks on the wheel. Then, since the work done is equal to the product of the mean moment of resistance, over the angle turned through, multiplied by the angle, if the resistance is constant over time, the moment of the *brake*, multiplied by the whole angle, measured the work done.

It is however to be noticed that the assumption that the *time-mean* of the moment on the brake is the same as would be the *angle-mean* of this moment might involve an error of any extent, provided the resistance and the angular velocity varied in conjunction. And as steam engines invariably exert an effort within the period of the revolution while the friction and the pressure causing it are apt to respond to any variations of speed, it is probable that there has been some error from this cause in all such measurements although not previously noticed.

HIRN appears to have been the first to recognise that in a steady condition the resistance of fluid between the brake-wheel and the brake would answer instead of the solid friction, so that the mean time moment of effort exerted in turning a paddle in a case with bafflers containing water would be strictly measured by the mean time moment of the case. And although subject to the same error from periodic motion as the friction brake, the facility this fluid brake offered for cooling and regulating led to its simultaneous adoption and development by several inventors, for measuring power—the late WILLIAM FROUDE, for the purpose of measuring the work of large engines, inventing that arrangement of paddle vanes and bafflers which gives the highest resistance, regulating the resistance by thin sluices between the vanes and bafflers, and always working with the case full of water.

The brake under consideration differs from that of Mr. FROUDE in only one fundamental particular—the provision by which a constant pressure in the interior of the brake is secured by the admission of the atmosphere to that part of the brake where the dynamical effect of the water is to cause the lowest pressure—this admits of working the brakes with any quantity of water from nothing to full, and thus allows of the regulation of the resistance by regulating the quantity of water in the brakes without sluices.

The description of this brake has already been published, together with that of the engines,* but it will be convenient to give a short description.

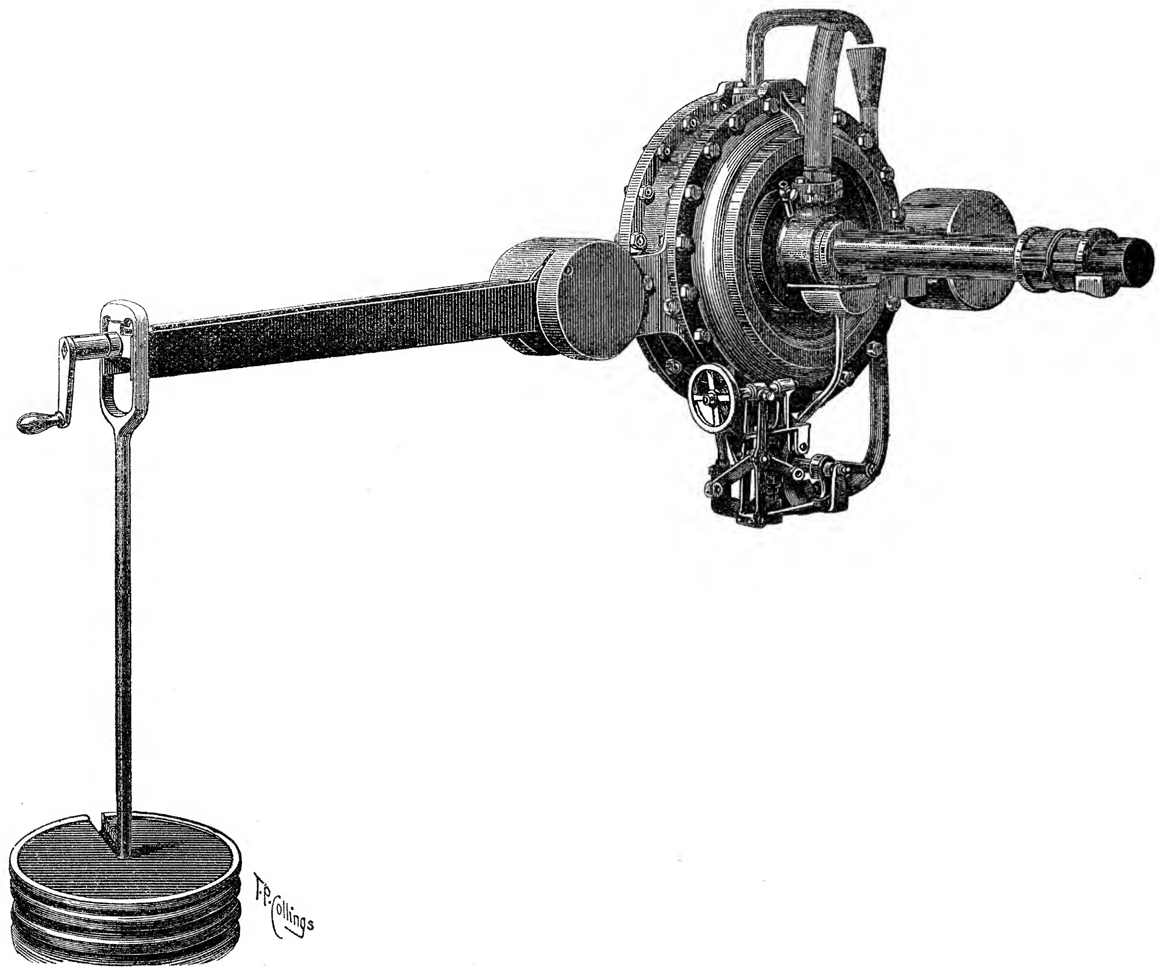
This brake consists primarily of (1) a brake wheel, 18 inches in diameter, fixed on

* "Triple Expansion Engines," by Professor OSBORNE REYNOLDS, 'Minutes of Proceedings, Inst. C.E.,' vol. 99, 1889, p. 18.

the 4-inch brake shaft by set pins, so that it revolves with the shaft (figs. 2 and 3), and (2) a brake (or brake case) which encloses the wheel, the shaft passing through *bushed* openings in the case which it fits closely, so as to prevent undue leakage of water while leaving shaft and brake-wheel free to turn in the case, except for the slight friction of the shaft (figs. 1, 2 and 3).

The outline of the axial section of the brake-wheel is that of a right cylinder, 4 inches thick. The cylinder is hollow—in fact, made of two discs which fit together, forming

Fig. 1.

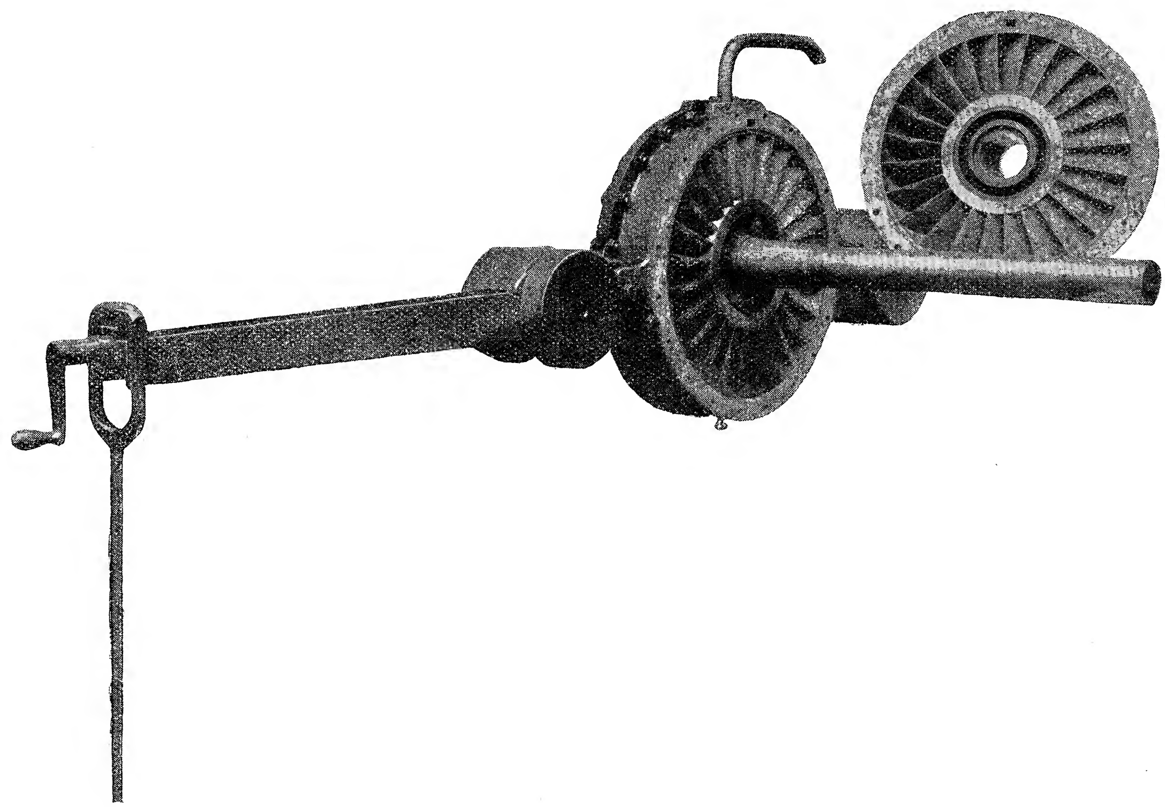


an internal boss for attachment to the shaft, and also meet together at the periphery, forming a closed annular box, except for apertures to be further described (fig. 3). In each of the outer disc faces of the wheel are 24 pockets (carefully formed), $4\frac{1}{2}$ inches radial and $1\frac{1}{2}$ inches deep measured axially, but so inclined that the narrow partitions or vanes ($\frac{1}{4}$ inch) are nearly semicircular discs inclined at 45° to the axis; the vane on one face being perpendicular to the vane on the opposite face (fig. 2).

The internal disc faces of the brake case, as far as the pockets are concerned, are the exact counterparts of the disc faces of the wheel (except that there are 25 pockets), so that the partitions in the case are in the same planes as the partitions meeting them in the wheel, there being $\frac{1}{64}$ inch clearance between the two faces.

The pairs of opposite pockets when they come together form nearly closed chambers, with their sections, parallel to the vanes, circular. In such spaces vortices in a plane inclined at 45° to the axis of the shaft may exist, in which case the centrifugal pressure on the outside of each vortex will urge the case and the wheel in opposite directions inclined at 45° to the direction of motion of the wheel,

Fig. 2.



which will give a tangential stress over the disc faces of the wheel of $1/\sqrt{2}$ of the sum of these vortex pressures. The existence and maintenance of these vortices is insured by the radial centrifugal force of the water in the pockets in the wheels owing to its motion.

This is the late Mr. W. FROUDE's arrangement. But an essential feature of the brake is the provision which insures the pressure of the atmosphere at the centre of the vortices, even when the pockets are only partially filled.

The vortex pressure is greatest at the outsides of the vortices, which occurs all over the annular surfaces of the pockets, but the actual pressure on these surfaces is

not determined solely by the vortex motion unless the state of pressure at the centre of the vortices is fixed, for the vortex motion only determines the difference between these pressures. To insure the constant pressure, and at the same time to allow of the pockets being only partially full—that is, to allow of hollow vortices with air cores at atmospheric pressure, it is necessary that there should be free access of air to the centres of the vortices, and as this access cannot be obtained through the water, which completely surrounds these centres, it is obtained by passages ($\frac{1}{8}$ inch diameter) within the metal of the guides, which lead to a common passage opening to the air on the top of the case (figs. 2 and 3).

To supply the break with water there are similar passages in the vanes of the wheel leading from the box cavity, which again receives water through ports which

Fig. 3.

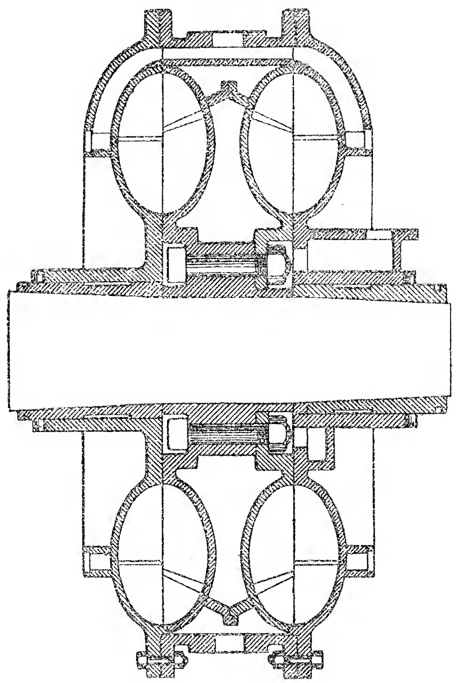
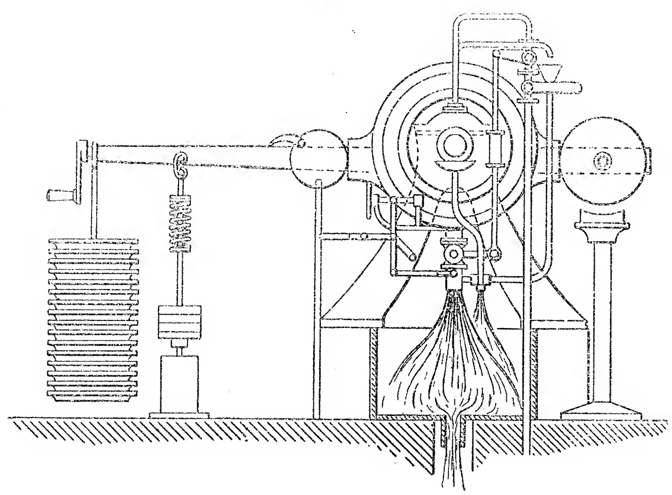


Fig. 4.



open opposite an annular recess in one of the disc faces of the case into which the supply of water is led, by means of a flexible indiarubber pipe from the supply regulating valve.

The water on which work has been done leaves the vortex pockets by the clearance between the disc surfaces of the wheel and case, and enters the annular chamber between the outer periphery of the wheel and the cylindrical portion of the case, which is always full of water when the wheel is running, whence its escape is controlled by a valve in the bottom of the case, from which it passes to waste.

By means of linkage connected with a fixed support and the brake case, an automatic adjustment of the inlet and outlet valves, according to the position of

the lever, is secured without affecting the mean moment on the brake case. And this also affords means of adjusting the position of the lever. To admit of adjustment for wear the shaft is coned over that portion which passes through the bushes, the bushes being similarly coned, and screwed into short sleeves on the casing, so that by unscrewing them the wear can be followed up and leakage prevented.

The brake levers for carrying the load and balance weight, are such as to allow the load to be suspended from a groove parallel to the shaft, at 4 feet from the shaft, by a carrier with a knife edge, the carrier and the weights each being adjusted to 25 lbs. (shown figs. 1 and 4). In addition to this load, a weight is suspended from a knife edge on the lever nearer the shaft, this weight being the piston of a dash-pot in which it hangs freely, except for the viscous resistance of the oil. This weight being adjusted to exert a moment of 100 ft.-lbs., and again a travelling weight of 48 lbs., is carried on the lever and worked by a screw with $\frac{1}{4}$ inch pitch, so that one turn changes the moment by 2 ft.-lbs., while a scale on the lever shows the position. A shorter lever on the opposite side of the case carries a weight of 74·6 lbs., which is adjusted to balance the lever and sliding weight when the load is removed.

The Accuracy of the Brake.

5. The principle of these hydraulic dynamometers is that when moment of momentum is introduced into a fixed space without altering the moment of momentum within that space, the rate at which moment of momentum leaves the space must equal the rate at which it enters. The brake-wheel imparts moment of momentum to the water within the case, and the friction of the shaft imparts moment of momentum to the case. The water in the case, when its moment of momentum is steady, imparts moment of momentum to the case as fast as it receives it, and the time mean of the moment of the load is equal to the time mean of the moment of effort of the shaft.

This is not affected by water entering and leaving the case at equal rates, provided it enters and leaves radially.

The condition of steadiness is, however, essential, in order that the moment of effort shall be at each instant equal to the moment of resistance on the case; any change in the moment of momentum of the water in the case being the result of the difference of the moment of effort on the shaft and that of resistance on the case.

The Time-Mean of the Moment of Effort.

6. When, however, the shaft is run over an interval of time, the mean moment of resistance on the case, less the difference of the moments of momentum of the water, at

the end and beginning of the interval, divided by the time, is the time-mean moment of effort on the shaft.

The possible limit of this error may be estimated when the maximum moment of momentum of the water is known as well as the minimum moment of resistance, and the minimum interval of time.

Thus taking the limits to be 30 lbs. of water, with radius of gyration 0·66 foot at 300 revolutions a minute (< 14), the interval of running 3600 seconds, the moment of the load 400 ft.-lbs., the limit of the time-mean of change of moment of momentum of the water is 14/3600, and this divided by the mean moment of resistance gives as the limits of relative error, $\pm$ 0·00001. This is supposing the whole of the water to be absent at the beginning or end of the trial, while the actual difference never amounts to more than 2 or 3 lbs., so that the limits do not exceed 0·000001, which is neglected.

The Angle-Mean of the Moment of Effort on the Shaft.

7. As already pointed out in Art. 4, when both the angular velocity of, and the moment of effort on, the shaft are subject to fluctuations of speed, the time-mean of the moment of effort may differ from the angle-mean. This applies to all brakes, but in hydraulic brakes, in which the resistance is proportional to the square of the speed although lagging by an unknown interval, it becomes possible to estimate the possible limits of this error when the limits of fluctuation of speed are known.

Taking ω the angular velocity of the shaft and ω_0 the time-mean of the angular velocity, $2a^2\omega_0$ the extreme differences of speed, and assuming the variation to be harmonic,

$$\omega = \omega_0 \{1 + a^2 \cos n (t - T_1)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

$$\omega^2 = \omega_0^2 \{1 + \frac{a^4}{2} + 2a^2 \cos n (t - T_1) + \tfrac{1}{2}a^4 \cos 2n (t - T)\} \quad . \quad . \quad (2).$$

Then to a second approximation neglecting a^6, if T_2 is the interval of lagging in the resistance and M the moment of resistance at the time t,

$$M = M_0 \{1 + 2a^2 \cos n (t - T_1 - T_2) + \tfrac{1}{2}a^4 \cos 2n (t - T_1 - T_2)\} \quad . \quad (3),$$

where M_0 is the time-mean of the moment of resistance. Also the rate at which work is done with uniform velocity, is $M\omega_0$, of which the mean is $M_0\omega_0$, and is the rate of work as measured by the mean moment on the case, multiplied by the mean-angular velocity.

To a second approximation the rate of work with varying speed is

$$M\omega = M_0\omega_0\{1+2a^2\cos n\,(t-T_1-T_2)+\tfrac{1}{2}a^4\cos 2n\,(t-T_1-T_2)\}\{1+a_2\cos n\,(t-T_1)\} \quad (4),$$

and from this it appears that the mean rate of work is

$$\omega_0 M_0\,(1 + a^4\cos nT_2),$$

which shows that the relative error in taking this as $M_0\omega_0$ is $+\,a^4\cos nT_2$. Thus the error arising from fluctuations in speed of $2a^2\omega$ is within the limits $\pm\,a^4$, when the resistance varies as the square of the speed, as in the hydraulic brakes.

Where, as in the brake under consideration, there is an automatic adjustment, by which the quantity of water in the brakes is adjusted to the speed, so as to maintain the resistance constant, there will be no error caused by such gradual variations of speed as result from changes in the boiler pressure, since the automatic adjustment can keep pace with them. But it takes time for the water to get in and out, and any variations, so rapid that, owing to the inertia of the brake case with its load, their effect has been reversed before the case has moved sufficiently to affect the water in the brake, will produce errors.

Such cyclic variations of speed attend all motions derived from reciprocating engines, and it is only these, and not the secular variations, that produce errors.

The Variations in the Speed of Rotation of the Steam Engine.

8. The cyclic variations all go through one or two complete periods in the time of revolution of the engine, and are approximately simple harmonic functions of the time.

They arise from three distinct causes :—

(1.) The varying energy of motion of the reciprocating parts ;
(2.) The varying moment of the effort of the steam pressures on the cranks ;
(3.) The effect of gravitation on the unbalanced parts in the engine.

In the case of a simple vertical engine, unbalanced and working with moderate expansion, these variations of speed may be severally estimated when I, the moment of inertia of the revolving parts, r the half-stroke of the reciprocating parts, and W the weight of these parts are known together with N the number of revolutions per minute, and U the work done per stroke.

For, considering the variations as existing separately, we may assume that the angular motion would be steady but for the particular effect, thus :

(1.) The moment of effort on the crank being constant, and the resistance constant, and equal to the effort, the energy of motion of all the parts is constant.

Putting $\omega = 2\pi N/60$, and $i = r^2W/g$,

$$\tfrac{1}{2}I\omega^2 + \tfrac{1}{2}i\omega^2 \sin^2 nt = C,$$

where C is constant, t is the time since the axis of the crank-pin has crossed the axis of the cylinder and n is ω_0, the mean value of ω or $2\pi N/60$.

Whence neglecting i as compared with I, the extreme variation of ω is approximately

$$2a_1^2\omega_0 = \tfrac{1}{2}\frac{i}{I}\omega_0$$

whence

$$a_1^2 = \tfrac{1}{4}\frac{i}{I}.$$

(2.) In the same way, considering the effect of the crank effort alone, with a moderate expansion, the energy that has to be absorbed and given out by the revolving parts is about one-fourth part of the work per stroke, and

$$\tfrac{1}{2}I\omega^2 - \tfrac{1}{8}U\cos 2n(t - T) = C,$$

where nT, say $\frac{\pi}{3}$ is the angle of the crank at which ω^2 is a minimum.

The extreme fluctuations in velocity are

$$2a_2^2\,\omega_0 = \frac{U}{4}\frac{\omega_0}{I\omega_0^2}, \quad a_2^2 = \tfrac{1}{8}\frac{U}{I\omega_0^2},$$

$$\omega = \omega_0\left\{1 + \frac{U}{8I\omega_0^2}\cos 2\left(nt - \tfrac{1}{3}\pi\right)\right\}.$$

(3.) The effect of the weight of the reciprocating parts acting alone, causes a fluctuation on the revolving parts of $2rW$; thus approximately

$$\tfrac{1}{2}I\omega^2 - rW\cos nt = C,$$

and

$$\omega = \omega_0\left(1 + \frac{Wr}{I\omega^2}\cos nt\right)$$

giving an extreme fluctuation on the angular velocity of

$$2a_3^2\omega_0 = 2\frac{Wr}{I\omega^2}\omega_0.$$

The equation of velocity is thus approximately expressed by

$$\omega = \omega_0 \left[1 + \tfrac{1}{4} \frac{i}{\mathrm{I}} \cos 2nt + \frac{\mathrm{U}}{8\mathrm{I}\omega^2} \cos 2\left(nt - \tfrac{1}{3}\pi\right) + \frac{\mathrm{W}r}{\mathrm{I}\omega^2} \cos nt \right].$$

In the low-pressure engine used in these experiments, the values of the several quantities are, the units being linear feet, lbs, seconds.

$$\mathrm{I} = 126, \qquad i = 2{\cdot}47, \qquad r = 0{\cdot}625, \qquad \mathrm{W} = 200, \qquad r\mathrm{W} = 125, \qquad \mathrm{U} = 1650,$$

$$\tfrac{1}{4}\frac{i}{\mathrm{I}} = 0{\cdot}0049, \qquad \frac{\mathrm{U}}{8\mathrm{I}\omega^2} = \frac{148}{\mathrm{N}^2}, \qquad \frac{r\mathrm{W}}{\mathrm{I}\omega^2} = \frac{90}{\mathrm{N}^2},$$

whence, substituting

$$\omega = \omega_0 \left(1 + 0{\cdot}0049 \cos 2\omega_0 t + \frac{148}{\mathrm{N}^2} \cos 2\left(\omega_0 t - \frac{\pi}{3}\right) + \frac{90}{\mathrm{N}^2} \cos \omega_0 t \right),$$

from this the approximate joint error can be found. But it is sufficient here to show that the individual errors are negligible.

The first gives an error in the mean moment

$$\pm \mathrm{M}\,(a_1{}^4 < 0{\cdot}000024).$$

The second and third are inversely proportional to N^4, if N is 300, which is the lowest value.

The second error is between

$$\pm \mathrm{M}\,(a_2{}^4 < 0{\cdot}0000025).$$

The third

$$\pm \mathrm{M}\,(a_3{}^2 < 0{\cdot}0000001).$$

These are all negligible quantities, and, as the corresponding effects in the high-pressure and intermediate engines, owing to the cranks being set at angles of 60°, would only be to compensate those of the low-pressure engine, the greatest error would not exceed $\frac{1}{40000}$th part.

9. Besides the errors resulting from the terminal differences in the moment of momentum of the water and the fluctuations of speed in the engine, error in the measurement of the work may arise from imperfect balance of the brake, from the frictional resistance of the automatic gear, from unequal resistance in rising and falling of the piston of the dash-pot, and from the end oscillation of the brake.

The Error of Balance of the Brake.

Although, when the shaft is running, the brake levers are perfectly free between the stops, yielding to the slightest force even when carrying a load of 400 pounds in addition to the weight of the brake-case of over 300 pounds, yet, when the shaft is standing, it requires a moment of some 40 ft.-lbs to move the lever in either direction, so that the balance can only be obtained as the difference of these moments, and this can only be obtained to about 1 foot pound. But, it is to be noticed that as long as the distribution of weights are unaltered and the lever is in the same position, any error of balance, whatever might be its cause, would be the same for all trials, no matter what might be the difference in the suspended load; so that, in taking the difference of the trials, the error would be eliminated, and, to insure this, the automatic adjustment was so arranged that, by a screw adjustment, the lever could be raised or lowered without affecting the automatic adjustment of the valves (see fig. 4, p. 308). Also an index was arranged adjacent to the end of the lever to which it might be always adjusted (shown in Plates 4 and 5).

The Error of Balance resulting from Friction of the Automatic Gear,

This had been a matter of serious consideration in designing the brakes, for, although it was obviously possible to so balance the parts of such gear that there should be no pressure against the fixed support arising from the weight of this gear, it was not obvious that the friction of these valves and their gear would not allow of a steady resistance to motion being maintained—would not allow the brake to lean against the fixed support within the limits of friction. However, after careful consideration of various contrivances, I came to the conclusion that, if the gearing between the support and the valve were inelastic, the joints being an easy fit, the tremor of the shaft and the brake, when running, might be depended upon to release any frictional resistance in this gear; so that, after any change, the gear would rapidly return to equilibrium. This proved to be the case, even to an unexpected extent, as was shown by the freedom of all the pins.

It was subsequently found by experiment that, even when the valves were so tight that it required a moment of 30 ft.-lbs. on the brake to move the automatic gear alone, with the shaft standing, in either direction, when the shaft was running any tendency to lean upon the support in either direction was the result of imperfect balance in the gear; and that, by adjusting this balance to an extent which would not cause a moment on the brake of 0·01 ft.-lb., the tendency of the brake to lean either in one direction or the other might be reversed—showing that, with a load of 600 ft.-lbs., the relative limits of error are $< \pm 0{\cdot}000016$, and in the difference of the trials would be zero.

The Work done in the Brake by End Play in the Shaft.

The clearance in the brake-case would allow of nearly $\frac{1}{32}$-inch end play along the shaft; and when the brake is running, owing to the slight *end*-play of the engine-shaft, there is at times a slight backwards-and-forwards movement, in the period of the engine, of the brake-case on the shaft, but not more than the 64th of an inch at the greatest. This end play, when it existed at 300 revolutions and 1200 ft.-lbs. load, could always be prevented by an end pressure on the case of < 50 lbs. Hence the limit of work done on the brake is $< 2 \times 50 / 12 \times 64 = 0{\cdot}13$ ft.-lb., which, compared with the work in one revolution with a load of 1200 ft.-lbs., is $0{\cdot}13/1200 \times 2\pi = 0{\cdot}000017$. This would be the limit if the error is proportional to the load, while, if constant, the error on the difference of two trials would be zero; so that the greatest relative error is less than

$$+ 0{\cdot}000017.$$

The Error from the Dash-Pot.

Since the piston is suspended freely in the oil-cylinder, and the resistance of the oil is viscous and expressed by $\mu vs / a$, where μ is the coefficient of viscosity, v the velocity of the piston, s the area of surface, and a the distance between the surfaces, the total resistance is thus $\mu s/a$ multiplied by the total displacement (which never exceeds 0·1 ft.) divided by the time (3600 seconds). This is infinitesimal. Besides which, with 1200 or 600 ft.-lbs. load at 300 revolutions, the lever remains perceptibly steady, there being no vertical vibration perceptible to the finger on the lever. Hence, as long as there are no oscillations, the limit of error from the dash-pot, if any, is imperceptibly small.

The only circumstances under which the lever oscillates is when the water flowing through is less than about 4 lbs. a minute; then a slow oscillation appears, the lever moving some half-inch, which causes the automatic gear to lean on the fixed support, and may cause a small error.

The Development of the Thermal Measurements.

The appliances were originally designed, in 1887, solely for the purpose of the study of the action of steam in the engines, and certain problems in hydraulics and dynamometry, without any intention of their being used for the purpose of measuring the heat equivalent of the work absorbed, but rather the other way.

It was, of course, obvious that, as the primary purpose of the brakes was to afford accurate measurement of the work spent in heating water, it was only necessary to measure the change of temperature of the water between entering and leaving the

brake, as well as its quantity, to obtain an approximate estimate of the heat equivalent of the work done. But the recognition of the extreme difficulty of obtaining any first-hand assurance as to the accuracy of scales of thermometers, and the fear of creating erroneous impressions as to the value of the equivalent, made me reluctant to allow such determinations. For this reason, as well as to avoid complicating the brake, in the first instance I made no provision for the introduction of thermometers, as may be seen in Plate 3.

But, after the engines and brake had been in use for two years, and had been found to possess attributes in steadiness of running, delicacy of adjustment and balance, beyond what I had dared to expect, and particularly in being able to work with an almost absolutely steady supply of water between steady temperatures, and the same temperatures for different powers, arising either from differences of speed, or differences of load, I realized that by working with the same thermometers on the same parts of their scales, and with the same loads and temperatures at different speeds, since the relative error of balance would be the same, if the surrounding temperatures were the same, the difference of two trials would afford the means of determining the loss of heat by radiation, and, this being determined, the difference of two trials made at the same temperatures as the previous trials, and both at the same speeds, but with different loads, would afford data for determining the error of balance without introducing the value of the equivalent or the use of the scales of the thermometers, except to identify equal temperatures.

I then yielded to the very general wish on the part of those who worked in the laboratory, and added such provision to the brake on the low-pressure engine as would admit of the measurement of the heat carried away by the effluent water, but only for the purpose of verifying the accuracy of balance as determined by mechanical means.

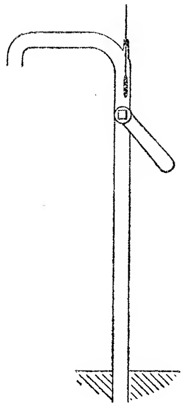
The Thermal Verification of the Balance of the Brakes.

10. The desirability of such independent determination of the balance arose in the first instance from the circumstances already described (Art. 9), viz., that the statical balance could only be determined to 1 ft.-lb., while the absence of effect from the friction of the automatic gear, &c., was only arrived at by somewhat complicated considerations.

The supply of water to the brake came from the service tank, 10 feet above the floor, and 7 feet above the shaft, the tank being supplied direct from the town main, and regulated by a ball-cock. The pipe from the tank passes beneath the concrete floor to a point conveniently close to the brake, whence a branch, in which is a hand-cock, rises vertically to a height of 4 feet above the floor, at which height is the automatic inlet valve, and from this the pipe is bent over, so that its mouth is directly over the inlet opening into the brake, with which the pipe is connected by a flexible indiarubber tube.

The first provision made for measuring the temperature of the entering water was an opening in the bend of the pipe over the inlet valve, with a vertical $\frac{3}{8}$-inch brass tube soldered in, about 4 inches long. This admitted of an indiarubber cork, through the centre of which a thermometer was passed into the pipe, as shown in fig. 5. This was afterward replaced by a glass thermometer chamber, as shown in the figure, Plate 5.

Fig. 5.



To measure the temperature of the water leaving the brake it was necessary, by means of a pipe fixed to the mouth of the outlet valve, to bring the effluent water above the balancing lever of the brake, and to one side of it. This pipe was arranged so as to admit the introduction of a vertical thermometer into the ascending pipe, much in the same way as the other. In the first instance the extension passage and the thermometer were all rigidly attached to the brake, and moved with it, which entailed a re-balance of the brake. Subsequently another arrangement was made. The thermometers used were divided to one-fifth of a degree Fahrenheit; they were both immersed in the flowing water to within a few degrees of the top of the mercury. They were compared at equal temperature, but otherwise subjected to no tests for accuracy of scale.

In making the experiments the link connecting the inlet valve with the automatic gear was removed and the valve was set open, the supply being adjusted by the hand-cock below. The head on the inlet being constant, when the cock was set the flow was practically steady. The quantity of water in the brake then depended on the outlet valve, which, with the exception of a little trouble at starting and stopping, soon overcome, kept the brake lever steady.

To catch the water after leaving the outflow thermometer, the extension pipe turned horizontally over the lever and then turned downwards into a basin, the lip of which was above the mouth of the pipe, and from the basin flowed in a short trough, from which it was caught in buckets. In these it was taken to the scales and carefully weighed. This was a primitive arrangement, and required several assistants, but was found capable of considerable accuracy up to about 40 lbs. a minute.

In making these experiments the engines were kept running at nearly constant speed by keeping constant pressure in the boiler. The speed being indicated on the speed gauge as well as recorded on the counter.

The water entering the brake, coming, as it did, from the town's main, was at nearly constant temperature between 40° and 50° Fahr., according to the time of the year, and varying less than a degree throughout several trials.

The rise of temperature was adjusted by the quantity of water admitted, according to the work, so that the final temperatures as well as the initial were as nearly as possible the same in the different trials.

This rise was such as admitted of the temperature of the brake being the same as that of the laboratory, which could always be adjusted to about 70° Fahr., so that the rise was from 25 to 30 degrees. This, with 40 lbs. a minute, required from 25 to 30 h.p.

Before commencing the actual trial everything was adjusted, and the engines running with steady load and steady speed until the thermometer showed the heat to be steady at the desired temperature, then, at a signal, the counter was put in and the water caught, each of the thermometers, and one giving the temperature of the laboratory, being then read at minute intervals over 15 or 30 minutes, when, on a signal, the counter was removed and also the last bucket.

The results of these tests were very consistent, within about 0·3 per cent., which was within the limit of accuracy then aimed at.

Trials with equal loads and different speeds showed that the loss by radiation was very small, while those at the same speed with different loads showed the balance was within the limits determined by mechanical tests.

In these trials the only correction was that for the lubricating water which escaped from the brake bushes. This was caught at each bearing, and the temperature taken so that the heat might be added, this being seldom more than 3 per cent. It may also be noticed that in these trials the heat lost or gained by conduction to or from the shaft was included in the radiation. As the brake is on an overhanging shaft which extends no farther than the outer bush of the brake case (Plate 3), the only conduction is on the side at which the shaft is continuous, where the brake bush is only some 4 inches from the brass of the shaft bearing. As the temperature of the brake on this side, which is opposite to that at which the cold water enters, was kept by the lubricating water at the temperature at which the water left the brake, and this was at temperature of the laboratory, there would be no cause of conduction unless the friction of the shaft in its bearing caused its temperature to rise above that of the laboratory. When the lubrication was good this was small, although on one or two occasions it made itself felt.

The Idea of Raising the Temperature from 32 *to* 212.

11. These tests became an annual exercise in the laboratory, and a very instructive exercise. But, as the subject—the value of the equivalent—was attracting much attention, the desire to obtain measures of it from these trials, by those engaged in them, resulted in Mr. T. E. STANTON, M.Sc., then Senior Demonstrator, effecting, for his own satisfaction, a comparison of the scales of the thermometers used in the experiments with a thermometer used in the Physical Laboratory, which had been compared with the air thermometer, and introduced these corrections into the results of the trials, which so gave values very close to what might be expected. I could not see however that determinations made with thermometers so corrected

could have any intrinsic value, but, as the matter was exciting great interest in the laboratory, I carefully considered the conditions which would be necessary in order to render the great facilities, which this brake was thus seen to afford, available for an independent determination.

The institution of an air thermometer was carefully considered and rejected. But it occurred to me that it might be possible to avoid the introduction of scales of the thermometers, just as before, and yet obtain the result. If it could be so arranged that the water should enter the brake at the temperature of melting ice and leave it at the temperature of water boiling under the standard pressure, all that would be required of the thermometers would be the identification of these temperatures. At first the difficulties appeared to be very formidable. But on trying, by gradually restricting the supply of water to the brake when it was absorbing some 60 h.p., and finding that it ran quite steadily with its automatic adjustment till the temperature of the effluent water was within 3° or 4° of 212° Fahr., I further considered the matter and formed preliminary designs for what seemed the most essential appliances to meet the altered circumstances.

These involved—

(1.) An artificial atmosphere, or a means of maintaining a steady air pressure in the air passages of the brake of something like one-third of an atmosphere above that of the atmosphere.

(2.) A circulating pump and water cooler, by which the entering water (some 30 lbs. a minute) could be forced through the cooler and into the brake, at a temperature of 32°, having been cooled by ice from the temperature of the town main.

(3.) A condenser by which the effluent water leaving the brake at 212° Fahr. might be cooled down to atmospheric temperature before being discharged into the atmosphere and weighed.

(4.) Such alteration in the manner of supporting the brake on the shaft as would prevent excess of leakage from the bushes in consequence of the greater pressure of the air in the brake, since not only would the leaks be increased, but when the rise of temperature of the water was increased to 180°, the quantity for any power would be diminished to one-sixth part of what it would be for 30°, so that any leakage would have six times the relative importance.

(5.) Some means which would afford assurance of the elimination of the radiation and conduction, as, with a rise of 140° Fahr. above that of the laboratory, these would probably amount to two or three per cent. of the total heat.

(6.) Scales for greater facility and accuracy in weighing the water, with a switch actuated by the counter.

(7.) A pressure gauge or barometer, by which the standard pressure for the

boiling point might be readily determined at 3° or 4° Fahr. above and below the boiling point, so as to admit of the ready and frequent correction of the thermometers used for identifying the temperature of the effluent water.

(8.) Some means of determining the terminal differences of temperature and quantity of water in the brake, which would be relatively six times larger with a rise of 180° than with 30°.

The Special Appliances and Preliminaries of the Research.

12. Having convinced myself by preliminary designs, not only of the practicability of the appliances, but also of the possibility of their inclusion in the already much occupied space adjacent to the brake, there still remained much to be done in the way of experimental investigation to obtain data from which the requisite proportions of these appliances could be determined, and these preliminary investigations were not commenced till the summer of 1894, when Mr. MOORBY undertook to devote himself to the research.

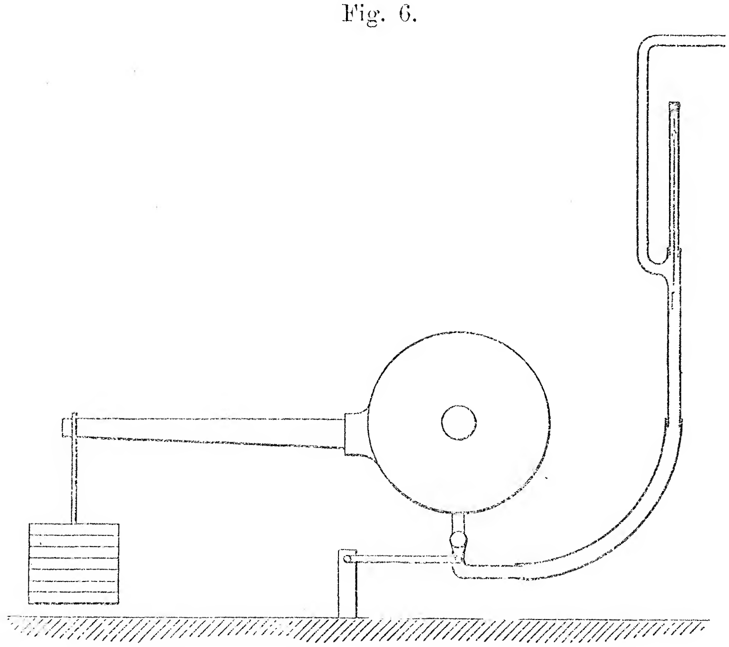
Weighing Machine and Tank.

13. The first step consisted in obtaining a somewhat special table weighing machine (Plates 4 to 6), having two rider weights on independent scales, one divided to 100 lbs. from 0 to 2200, the other to 1 lb. from 0 to 100. Also a galvanized iron tank, $5' \times 2'\,9'' \times 2'\,9''$, capable of holding above one ton of water, with a 4-inch screw valve at the bottom, opening inwards by a handle above the top of the tank, the top of the tank being covered with carefully fitted, but separate, $\frac{1}{2}$-inch pine boards, previously steeped in melted paraffin-wax, to prevent adhesion or absorption of water. This machine and tank, which is a large affair, was placed in the only position available, opposite the end of the shaft and behind the standing pipes for supplying the condensing water to the engine, thus leaving the passage between these pipes and the end of the shaft open, an important matter, as this passage was the only place from which the observations on the brakes could be made. This entailed the carrying the outflow from the brake over the passage, about 6 feet 6 inches from the floor.

Design of the Outflow.

14. This extension of the pipe further entailed the necessity of making this pipe a fixture, and connecting it with the outlet below the automatic cock by a *bent* wire-bound flexible indiarubber pipe, so as to prevent any moment on the brake. (See fig. 6.)

Fig. 6.



The Thermometer Chambers.

15. A glass chamber for the outflow thermometer was introduced as shown (fig. 6), and another for the inlet, somewhat similar. These were arranged so that the bulbs of the thermometers were down in the full current while the scale was in the glass tube, through which a portion of the water was allowed to flow, that from the inlet thermometer being conducted away to waste, while that from the outlet was conducted back again into the outflow pipe. In this way, not only the bulbs of the thermometers, but the entire thermometers were immersed in the flowing water.

The Two-way Switch.

16. A switch, as shown in Plate 5, was also constructed for diverting suddenly the stream of effluent water from waste to the tank, or *vice versâ*, without exposing the stream for more than an inch, and without any splashing or uncertainty.

Experience in Making Observations.

17. When these arrangements were completed, and whilst the other appliances were progressing, Mr. Moorby commenced a series of experiments similar to those which had been previously made, using the water from the tank at the temperature of the town's water, and raising it to temperatures which were successively increased. This was with a view of testing the improved facilities, and also of gaining experience and facility in making and recording the observations.

The engines and brakes were occupied two or three times a week in the ordinary work of the laboratory, so that there were only one or two days a week available for these experiments, and every opportunity was valuable.

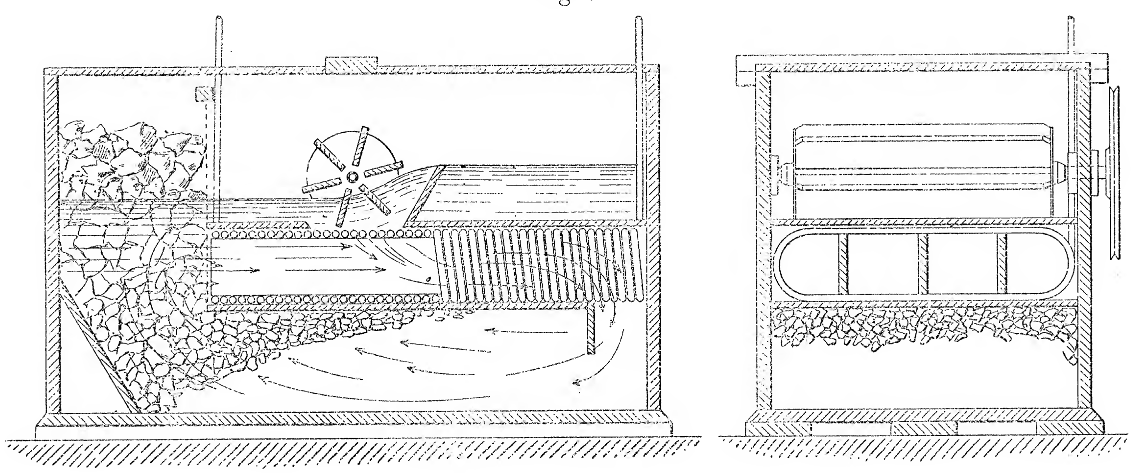
The Design of the Condenser.

18. At the same time he made experiments to determine the necessary length of pipe in order that the water flowing along it at the rate of 20 lbs. a minute would be cooled from 212° to 70°, when the pipe was jacketed by a stream of town's water at 50° Fahr.; by the result of which experiments the condenser in which the effluent water is cooled to 75° was designed (Plates 4 to 7).

Design of the Ice-Cooler.

19. To cool the water to 32°, or as near as practicable, I had, on account of the danger of some ice being carried through with the water if the ice were once put into the water, decided to pass the water through a long coil of ordinary water piping, immersed in water, towards the top of a tank with ice under the coil,

Fig. 7.



and from experiments made by Mr. Moorby, I decided on the coil and arrangements shown. The coil consists of $\frac{3}{8}$-inch composition pipe, 200 feet long, the tank being 2 feet 6 inches wide and deep and 4 feet long, the coil being placed near the surface of the water on a shelf with a wire netted space at the end for the introduction of the ice, which is pushed down under the shelf, and with a paddle which is kept in continual motion by a cord from the line shaft, thus securing a rapid circulation of the water. The tank is constructed of 1-inch pine saturated with paraffin wax, in preference to a metal tank.

In this design account had to be taken of the requisite head of water necessary to force some 20 lbs. a minute through the coil. It was estimated that this would require some 30 lbs. on the square inch, which, together with the 5 lbs. excess of pressure in the brake above the atmosphere, and a margin of some 25 lbs. in order to secure steadiness of flow, made a total of 60 lbs. on the square inch, or 122 feet of head.

The Circulating Pump.

20. It was essential that this head should be approximately steady, and under control during the trials, also that the water should be drawn as directly as possible from the town's mains, in order to secure both the low temperature and great purity of this water. This precluded the direct use of the water from the large tank in the tower, which would otherwise have just afforded this head. It also precluded the use of such head as there might be in the town's mains, as this was insufficient and continually varying, so that some special means of imparting the steady head to the water after drawing it from the tower mains was necessary. This involved pumping the water through the ice-cooler and brake. It might be done by pumping it from the service tank in the laboratory into an accumulator under a constant load, or by passing the water through a centrifugal pump, running at a steady speed on its way to the brake.

The facilities in the laboratory decided this question. There already existed the quadruple vortex turbine, with four three-inch wheels in series, worked from the water in the tower, which would work steadily up to 1 h.p., in a position which would be convenient for driving a centrifugal pump in the in-circuit of the pipe leading to the brake; I also had a quintuple centrifugal pump with five $1\frac{1}{2}$-inch wheels in series which was adapted to the purpose. It was decided, therefore, to lead the water from the surface tank, 9 feet above the floor, into the quintuple pump, driven by the turbine under a constant and controllable head, so that the head would be raised to the required amount. Then, to lead the water through the cooling coils to a pressure gauge close to the brake, and thence through a regulating valve into the passage with the thermometer leading into the brake. (See Plates 6 and 7.)

The Outlet from the Condenser.

21. In order to prevent the formation of steam, owing to the presence of air in the water, before it had passed the outlet thermometer, it was necessary to maintain a certain pressure in the effluent water as it passed the bulb of this thermometer. At first it was thought that a head of 5 or 6 feet would suffice. In order to secure this, the level of the condenser being some 3 feet above the bulb, the pipe leading from the condenser was carried up vertically about 3 feet higher, then turned over and led down again to an orifice immediately over the switch, while from the top of the bend a vertical branch extended upwards about 3 feet, with its mouth open, to the air. This was subsequently raised. (See Plate 4.)

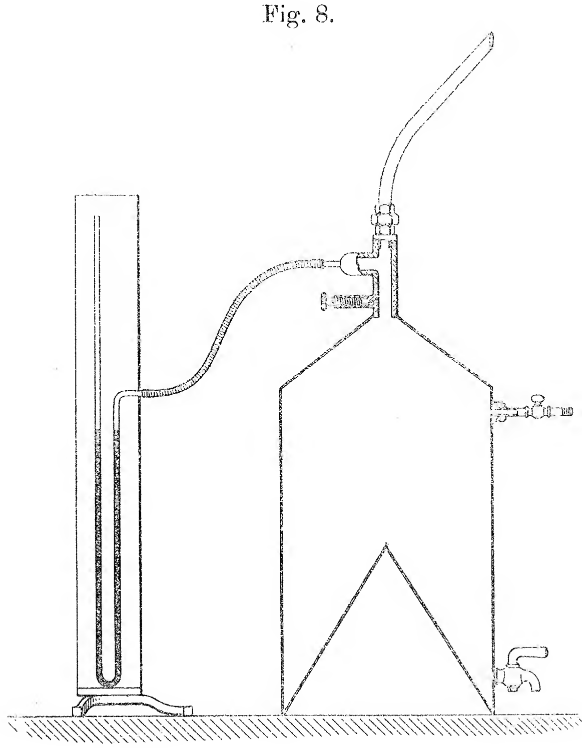
Preliminary Experiments at 212° Under Pressure.

22. The preliminary investigations and the construction of the appliances so far described, were not completed till May, 1895. It then became possible to make some experiments as to the working of the brake under pressure and at high temperature, so as to obtain guidance as to the artificial atmosphere and means of controlling the leakages at the bearings. From these experiments two things came out clearly. It was found that all that was necessary for an artificial atmosphere was to connect the outlet of the air passage on the top of the brake by means of a flexible india-rubber pipe capable of bearing the pressure to a vessel of very moderate capacity.

The Artificial Atmosphere.

23. A tin can, holding about 3 gallons, with the bottom and top coned upwards, and strong enough to stand the full pressure of 60 pounds, was adopted. The air connection with the can was at the top, at which there were also two side openings, one with a cock, to admit of air being pumped into the can, and the other with a fine

Fig. 8.



screw stop for allowing a slow and definite escape of air. An opening at the bottom, with a cock for drawing off water, was also provided. For forcing the air in, a syringe for inflating bicycle tyres was used in the first instance and proved ample; in fact, when once the pressure was raised, the small amount of air released from the water

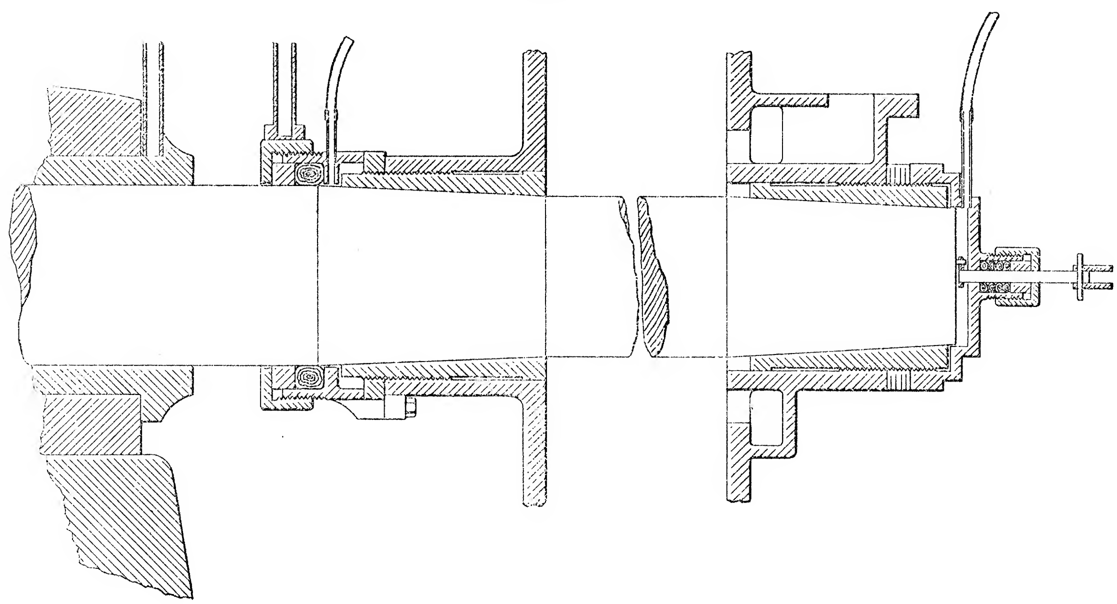
was more than sufficient to maintain the pressure, so that it was continually allowed to escape.

The Stuffing-box and Cap to prevent Leakage.

24. The thing that was revealed by the experiments at high temperatures was that the leakage of water at the coned bushes of the brake was so much increased by the pressure within the brake that even when these bushes were adjusted to run, as close as was practicable, on the cones of the shaft, this leakage was very considerable, so that some other method of controlling this escape became necessary.

This matter threatened to present great difficulties. It was apparently impossible to close in the bushes with stuffing-boxes and stop the leakage altogether, as that would prevent the lubrication of the shaft, and, apart from this, would cause the temperature on the shaft side of the brake to rise to the temperature of the brake, 212° Fahr., which would cause a large escape of heat along the shaft. Besides this, the adaptation of stuffing-boxes to the existing brake presented such difficulties that it almost seemed as though it would be necessary to have a new brake, which, besides the delay, would entail an addition of some £200 to the expenses, which were otherwise very considerable.

Fig. 9.



To avoid this I determined to try a stuffing-box on the shaft side, constructed in halves to be bolted together on the shaft, and then sweated into one, this stuffing-box to screw on to the exposed screw of the bush, and make a joint against the lock ring; then to open a passage through the box inside the packing-ring, with a tap to control the escape of water, and at the other end to screw a cap on to the bush, entirely inclosing the end of the shaft, with an aperture and a tap to regulate the water, also

a small stuffing-box in the cap, to allow of a spindle for connecting the shaft with the counter.

These entailed very difficult and exceptional work, but were beautifully executed by Mr. FOSTER, in the Laboratory (fig. 9).

However, the result was very doubtful, as the water flowing from the brake through the aperture in the stuffing-box not only raised the temperature of the shaft, but was itself of uncertain temperature.

It was in July, 1895, that this experience was obtained, and for a time the success of the research seemed doubtful. During the vacation, however, an idea occurred to me which at once promised to do away with the whole difficulty.

The Cooling and Lubricating of the Bushes.

25. This idea consisted of what seemed to be a practicable plan of forcing a relatively small, but sufficient portion of the ice-cold water into the brake through each of the bearings, the quantities being strictly under control.

That this plan should not have presented itself as soon as the addition of the stuffing-box and the cap were contemplated, becomes intelligible when it is remembered that the main object in the invention of this brake had been to secure a constant pressure in the air space within the vortices, so that by admitting the water through passages in the vanes directly into this air space a constant resistance, whether that of the atmosphere, or artificial atmosphere, on the entering water would be secured, and that the possibility of maintaining an even flow through the brake, so essential to any success in the research, depended entirely on the realization of this constant resistance. Except the inlet passage, the interior of the wheel, and the air space in the vortices, all the spaces in the brake and brake-case are under the full vortex pressure, excepting where, as in the bush on the closed side of the brake, and that between the solid disc faces on the inlet side, the pressure is relaxed by the escape of the water. This vortex pressure depends on the load on the brake, and may be anything up to 25 pounds on the square inch greater than that in the air-cores. It thus seemed like starting *de novo* to interfere with this arrangement; and it was only when one came to realize that the possibility of preventing all leakage by the introduction of the stuffing-box and the cap had rendered it possible, by controlled subsidiary supplies under pressure, to reverse the flow of the lubricating water, and so to do away with leakage, and not only to secure lubrication, but also to cool the bushes, and then only after considering the amounts of water required, and the provision in the way of pumping appliances, separate supplies of water and thermometers, &c., that the altered facilities afforded by the circulating pump came to be recognized.

The By-channels and Regulator admitting Cooled Water to the Bushes.

26. Since the main supply must enter, as before, at the same pressure as the air within the vortices, while, in order to reverse the flow through the bushes, that entering the cap must enter at a little, but only a little, above that of the air within, while that entering on the brake side of the packing-ring in the stuffing-box must enter at any pressure up to 20 lbs., according to the load, above that of the air within, it was clear that there must be three supplies of water at different pressures under separate control; and it was equally clear that these supplies must all be at the same temperature.

Fortunately, the arrangements already made for the new supply afforded ready means of securing these conditions, as, in order to insure steadiness in the supply through the regulating valve, it had been provided, in arranging the pump, that there should be an excess of 20 lbs. on the square inch above that necessary to force the maximum water through the coil and to overcome the air pressure in the brake; also, as the regulating cock was only an inch or two from the thermometer chamber, the water would be subject to little heating by radiation after leaving the cock, while the effect of radiation to the by-channels would be of secondary importance, as it is eliminated with the rest of the radiation in the difference of the trials.

It thus became possible, by leading cooled water through two short by-branches, with separate regulators, from the supply pipe, before passing the main regulator, respectively into the aperture through the stuffing-box on the inside of packing-ring, and into the cap on the inlet end, to secure controlled inflows of ice-cold water between each of the bushes and the shaft, and so to adjust the temperature of the bearing and insure lubrication of the shaft (fig. 9).

In order to render such inflows steady and constant, it was desirable that the pressures before passing the regulator should be kept at a considerable and constant quantity above the vortex pressure in the brakes.

From the first preliminary trials made with the branches it appeared that the turbine and pump were capable of supplying sufficient pressure for this, so that the only additions necessary were the branches. These were made of $\frac{1}{4}$-inch brass pipe from the main pipe from the cooler as far as the branch regulators, and thence continued by $\frac{1}{8}$-inch indiarubber vacuum tube $\frac{3}{4}$ inch outside wrapped with tape. The branch regulators have cocks, with provision for fine adjustment, so that the very small quantities which passed might be definitely regulated to great nicety (Plate 5). With these it was found practical to maintain the temperature of the bushes from anything a few degrees above 32 to any required temperature.

It is to be noticed that the work done by pressure over and above the pressure p_a in the inlet thermometer chamber is that due to the difference between the pressure in the main pipe before passing the regulators and p_a, through whichever passage the water enters. And since in that water which passes into the thermometer chamber

through the main regulator this work has been converted into heat, and is measured as entering heat by the inlet thermometer, the assumption that the water through the branches enters at the pressure p_a, and the temperature given by the inlet thermometer, involves no other error than that resulting from radiation, which is constant for all trials, and is eliminated in the difference.

The Regulation of the Temperature of the Bushes.

27. In the preliminary trials this temperature was only ascertained by touch, and regulated so as to be as nearly as possible that of the laboratory, the branch cocks being set with a definite opening, and the excess of pressure maintained as nearly as possible constant, a plan which was found to give consistent results. But it also appeared that in order to maintain the same temperature in the stuffing-box for the large and small trials with the same pressure in the main pipe, it was necessary to open the branch cock wider in the large trials. This was to be expected from the greater vortex pressure in the large trials. And as owing to the greater resistance of the cooler in the large trials there was difficulty in maintaining a great excess of pressure over the vortex pressure, it was decided to run both large and small trials with the same setting of the cock, and the same head in the cooling pipe, keeping a record until some means was obtained of estimating the comparative slopes of temperature in the shaft in the large and small trials.

The Measurement of the Comparative Slopes of Temperature in the Shaft.

28. The desirability of some more definite knowledge of the slope of temperature in the shaft between the brass of the nearest shaft bearing and the stuffing-box was strongly felt, but it was not at first apparent how this might be done, the shaft being 4 inches in diameter and the gap between the end of the stuffing-box and the brass of the bearing being only 3 inches.

However, as it became more evident, with the branch cocks set at a constant opening and the same pressures in the supply pipe, that the temperatures in the stuffing-box were greater in the large than in the small trials, and that a small difference in the adjustment of the branch cock to the stuffing-box affected the apparent loss of heat to the extent of some 0·1 or 0·2 per cent. of the total heat, I determined to try and obtain some definite evidence of the relative slopes of temperature in the two trials, by measuring the relative temperatures of the brass and the stuffing-box as far as was practicable. For this purpose, I had thick brass tubes, radiating outwards, sweated on to the end of the stuffing-box to hold thermometers. Two such tubes were necessary on account of the screwing-up of the box, which had to be done whenever it began to leak ; and although this was not done during a trial, one tube would sometimes face downwards, which was inconvenient. In a similar manner

two tubes were attached, one to the top and one to the bottom brass of the bearing, holes being bored into the brass and the tubes screwed in. These tubes are shown in fig. 9.

In this way, with a thermometer in one of the tubes on the stuffing-box and one in each of the tubes on the bearing, although the thermometers might not give the actual temperatures of anything in particular, still the steadiness of the conditions of the brake warranted the conclusion that the differences in the readings of the thermometers would serve to identify similar conditions as to slope of temperature, and this turned out to be the case.

These thermometers threw a flood of light on to conditions which had before been hardly perceptible. Thus, after reading the thermometer during three large trials and three small trials, with the cocks set as before without having been displaced, and with the same pressures, it was found that the mean of the three large trials indicated 13° Fahr. greater slope from the stuffing-box to the brass than that indicated by the mean of the three small trials.

The Constants and Limits of Error of Conduction.

29. It thence became possible in the subsequent trials, by adjusting the cocks, to bring about a mean condition in which the mean slope in the large trials was the same as that in the small, and by comparing the mean results of those trials in which the difference of slope had been in one direction with the mean of those in which it had been in the opposite, to obtain a constant expressing the quantity of heat lost for each degree of the recorded slope.

These thermometers, read to 1° Fahr. 7 times during the trial of each sort, would give a limit of error of the $\frac{1}{7}$ of a degree which, taking 12 thermal units per hour as the loss per degree, would give as limit of relative error on 100,000 thermal units of, on one trial,

$$0{\cdot}00002,$$

and these being casual, when taken over 40 trials would be less than a millionth.

The Hand-Brake for Regulating the Speed of the Engines.

30. Although it had been found possible to maintain the speed of the engine constant within 2 or 3 per cent. when the engines were working with a considerable margin of pressure in the boiler, by maintaining the pressure in the boiler constant, the care and attention on the part of Mr. J. HALL, who had charge of the engine, became excessive when the engines were indicating over 80 h.p., particularly as he could not be attending to the fire and lubrication, and at the same time watching the speed indicator. To meet this difficulty, as there is no known automatic governor

which will regulate an engine working against a resistance which is independent of the speed, without fluctuations, I arranged a hand-brake to be applied to the rope pulley, 3 feet in diameter, on the brake shaft, by one of the assistants in the laboratory during the trial. The amount of power to be absorbed by this being less than 2 h.p. at the most, a $\frac{3}{8}$-inch cotton rope, with one end fast, passed round in one of the grooves of the pulley, the other end being attached to a spring balance, the position of which could be regulated with a screw, would answer the purpose (shown in Plate 5).

In this way, as the natural variations of speed of the engines are very slow, Mr. MATHEWS was able, after a little experience, to keep the speed to within something like one revolution, or 0·3 per cent.

The Corrections for the Terminal Heat of the Brake.

31. As the temperature of the effluent water could be continually regulated by regulating the supply of water to the brake, whatever might be the speed, the chief importance of keeping the speed regular arose from the errors (1) caused by small differences of temperature in the brake together with the water it contained at the commencement and end of the trial, and (2) by small differences in the weight of water in the brake at the commencement and end of the trial.

Such errors belong to the class of casual errors to be eliminated in the mean of a number of trials. Still, it seemed desirable to have some assurance that such elimination was effected, and, in order to obtain this, I proposed that the actual quantity of water in the brake for each of the loads used in the experiments should be determined experimentally at several speeds covering the range of variations likely to occur, and so to obtain a curve for each load, showing the water at each particular speed; this to be done by running the brake as in the trials, steadily, at a particular speed, the water passing as in the trial. Then, suddenly, by forcing down the lever, to close the automatic outlet valve, and, at the same time shutting the inlet valve and stopping the engines, and thus trapping the working charge of water in the brake. The water could then be drawn out and weighed.

Putting B for the capacity for heat of the metal of the brake, w for the weight of water, and T for the temperature observed on the effluent thermometer, the total heat in the brake is expressed by

$$(B + w)\,T^\circ,$$

and, if w_i, T_i° refer to the weight of water and temperature at starting, and w_f, T_f° to the corresponding quantities at the end of the trial, the correction which has to be subtracted from the heat observed is expressed by

$$(B + w_i)\,T_i^\circ - (B + w_f)\,T_f^\circ.$$

The Method of Conducting the Trials—Elimination of Radiation.

32. The entire system of working was designed to secure the most perfect elimination of radiation possible. Thus, it was arranged in the first place that the trials be made in pairs, one heavy trial and one light trial, made under circumstances as nearly similar as possible, except in respect of load and water. The loads in the first instance being 1200 and 600 foot pounds, and the quantities of water such that the final temperature should be as nearly as possible 212° Fahr., and, after the preliminary trials, 300 revolutions per minute was adopted as the speed for all the trials, 60 minutes as the time of running. The inlet and outlet thermometers to be read after the first minute, and every two minutes; also the temperature of the laboratory as shown by a thermometer in a carefully-chosen place. This temperature to be maintained as nearly constant as possible. The setting of the regulators during each trial to be recorded; also the pressure of the artificial atmosphere, and that in the supply pipe after passing the coil; and, subsequently, the reading of the thermometers in the stuffing-box and bearings taken every five minutes, and the speed gauge every two minutes. The observations and incidents being recorded by the rules in surveying, in ink, in a book, and distinct from any reductions. The initial and final reading on the scales and counter being included, as were also the initial and final readings of the inlet and outlet thermometers and speed gauge for the purpose of determining the terminal differences of the heat in the brakes.

As it was impossible to make trials simultaneously, and so secure similar conditions in the laboratory, it was at first arranged that the trials should be made in groups, including four pairs of trials.

The regular work in the laboratory monopolised the engines and brakes on all days in term time, except Mondays and Thursdays, so that the trials were confined to two days in the week. There was a certain likelihood of the state of temperature of the walls and objects in the laboratory being systematically different on the Mondays, after the laboratory had been without steam all Sunday, from what it would be on the Thursday, after the steam had been on for three days. And besides this, there would be a systematic difference in the temperature of all the objects during the first trial in the day, although the brake had been running for an hour before, from that which would hold in the following trials. In the first instance, therefore, it was arranged that a heavy and a light trial should be made on the same day, and a light and a heavy trial on the next available day, under as nearly similar circumstances as possible, except for the inversion of the order. Then again, a light and a heavy trial on the next day, followed by a heavy and a light on the following, so as to break the order and secure the same arrangement, in days of the week as well as in hours of the day, for the four light trials as for the four heavy trials.

As the results of any group of four pairs of trials would furnish a tolerably close approximation to the loss of heat by radiation, assuming this to be proportional to

the observed mean difference of temperature between the laboratory and the *brake*, it was easy to obtain an approximate constant, R, for radiation for each degree of difference of temperature, and so to introduce a correction, R $(T_2 - T_a)$, in each trial for the radiation resulting from the observed mean difference of temperature of laboratory and brake, $T_2 - T_a$.

These corrections would serve two purposes—first, affording a better comparison of the results of the separate trials for future guidance, and secondly, by recording the mean difference of temperature, would show how far the mean differences of temperature in the large trials had differed from those in the small trials, and thus how far the radiation had been eliminated.

Lagging the Brakes.

33. In order to obtain still more definite assurance as to the elimination, it was arranged that after consistent results had been obtained in several groups of four pairs of trials, as above, with the brake naked, the brake should be covered with non-conducting material, in the best way practicable, so as greatly to reduce the radiation, at the same time leaving it definite, and then similar trials should be run.

If the coefficient of radiation could in this way be reduced to one-fourth that of the naked brake, such error as there might be remaining in the mean results with the naked brake would be reduced to one-fourth with the lagged brake.

In this, however, there was danger of introducing errors of other kinds.

The non-conducting material would absorb heat slowly and take a long time to arrive at a state of equilibrium, and during the interval the rate of loss of heat from the brake would be irregular. The total error that could result from this cause would be the product of the specific heat of the material used multiplied by the weight, and again by the 75°, or the half of whatever was the difference in temperature of the brake and the air. This decided the choice of the material to include cotton wool. Two pounds of this would, if not too tightly pressed, cover the brake about $1\frac{1}{2}$ inches thick, and the total heat it would absorb would be less than 0·4 lb. of water raised from 32° to 212° Fahr., and would then be only 0·0008 of the heat generated by 30 h.p. in an hour, while it would reduce the radiation to about $\frac{1}{7}$. But as the cotton wool would gradually collapse if subjected to any elastic pressure, it was decided only to use this to such thickness as it could be protected by light cotton strings extending in axial planes round the brake, and to prevent absorption of moisture by the cotton wool, to cover it with thick anti-rheumatic flannel, about 1 inch to $1\frac{1}{2}$ inches in thickness, as shown in Plate 5, which would raise the capacity for heat of the entire lagging to about $\frac{1}{600}$ that of the heat generated in the small trials, and as the brake was kept at steady temperature for about one hour or more before the trial commenced, the actual differences would not exceed some one ten-thousandth part.

The Conduction by the Levers.

This lagging only extended over the body of the brake covering all the brass-work, leaving the levers and balance weights on the levers bare.

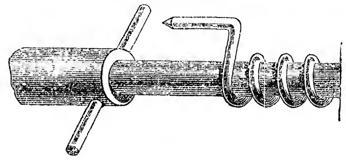
These levers being in metallic contact with the brass of the brake assumed at these points the temperature of the brake, and would conduct the heat along to the balance weights till it was lost by radiation. As the temperatures were constant in all the trials this loss of heat would merely form part of the radiation and be eliminated as the rest; but, owing to the masses of the balance weights and the length of the levers, it must take a long time for the balance weights and the further parts of the levers to arrive at a steady temperature, a fact which would account for a greater loss of heat in the first trial made in the day.

In order to obtain assurance that this delay produced no error it was arranged that after the completion of the series of trials with the brakes lagged, corresponding to that with the naked brakes, that the balance weights should be removed, and only the load at 4 feet from the brake left, and a third series of trials made.

Starting and Stopping the Trials.

34. Having adopted an hour as the length of each trial, and 300 revolutions as the normal speed, the engines having been running for an hour previously, while the water entering the brake was being adjusted, and afterwards, so as to insure the temperature, not only of the brake, but of the surrounding objects, having become approximately steady at the time of starting the trial, all that was necessary was that the counter should be pushed into gear, and at the same time the water-switch pushed over, and the reverse operation at the end of the trial. These operations, simple as they were, entailed errors, which arose partly from the impossibility of instan-

Fig. 10.



taneous engagement of the counter simultaneously with the switching of the water. In order to diminish these as far as possible, the spindle of a counter, on which was the worm which drove the worm wheel, was wrapped with a spiral spring of steel wire, which gripped the spindle so tight that it would not slip, the end of the wire being bent, so as to form a clutch standing off the shaft half-an-inch, the end of the wire being pointed, the shaft of the counter projecting a little beyond the wire. Facing the end of this shaft, and in line with it, was a socket in the end of the engine shaft,

which was brought down to three-quarters of an inch diameter and carried two round pins, a sixteenth of an inch diameter, standing out radially, the engagement being effected by pushing the counter forward till the wire crank engaged on one of the pins. (Owing to the wire being pointed and the pins rounded, the chance of the wire striking plumb on to the pin and so preventing engagement was reduced to a minimum.)

This engagement was the result of a great deal of experience, and answered perfectly, but it involved the mean chance of a quarter of a revolution of the engine shaft after the wire had passed the pin before the actual engagement was effected, whereas on coming off the disengagement was instantaneous, the counter stopping by the friction of the worm before the momentum had carried it through any appreciable angle.

This would leave a mean error of the work done during one-fourth of a revolution on each trial, whence, the number of revolutions during the trial being 18,000, the relative mean correction would be one seventy-two thousandth part, or 0·000013. As, however, when the two operations were executed by different observers on a signal, the personal equations might amount to more than this, although it involved a difficult piece of linkage, an automatic connection was effected, as shown in Plate 5, the pushing of the counter into engagement shifting the switch, so that in making the trials no error was introduced.

The Leakage of Water.

35. As the loss of any of the water, which had entered the brake before it was weighed, would constitute a corresponding error in the results, the perfect tightness, not only of all the fixed joints, but of the casting and the pipes, was a matter of first consideration and of continual care. This was one of the reasons why the lagging was delayed till after consistent results had been obtained; for, as long as the brake and pipes were naked, such leakage could not fail to be observed on close inspection, and before lagging it was arranged to test the brake and pipes to an excess of pressure, so as to insure perfect soundness. Besides the fixed joints there were only two working joints, in addition to the openings into the switch and again into the tank.

(1) The working joints were: The stuffing-box on the main shaft and the stuffing-box on the automatic cock on the outlet from the brake.

Any leakage from these was open to observation both before and after lagging, as they were in no way covered; and arrangements were made so that such leakage could be separately conducted by pipes and caught in bottles. With care such leakage could be reduced to insignificant quantities.

The absolute loss of heat resulting from a leakage of $w_{S.B}$ lbs. of water from the stuffing-box on the shaft was equal to the product of the difference of temperature of the stuffing-box $T_{S.B}°$, and inlet ($T_1°$) multiplied by $w_{S.B}$,

$$w_{S.B} (T_{S.B}° - T_1°),$$

and in the few trials in which this became a sensible quantity it was to be added as a correction.

The Loss of Heat by the Leakage of Water from the Automatic Cock.

36. This was the product of (w_c), the weight of water which escaped multiplied by the total rise of temperature. Since the water passing the cock was on its way to the high temperature thermometer, where any such water was caught it was put into the tank, and so required no correction. This leakage was very small, at most 2 oz. in a trial, but as there must be some evaporation as the water escaped through the hot gland, which, though small, might be of some importance on account of the latent heat of evaporation, it was desirable in some way to enclose this stuffing-box in an indiarubber bag closing on the spindle, so that the vapour could not escape, and this was eventually accomplished very effectively and neatly by Mr. FOSTER, in a way which did not interfere at all with the free action of the cock (Art. 14, Part II.).

The result of this, besides preventing any subsequent loss of water, in this way, was to show that any error that had previously existed from evaporation was inappreciable.

The Loss of Water at the Switch.

37. Apart from evaporation which would result from the exposure to the air and in passing the air-gap into the switch, there was no loss, as the water descended almost tangentially on to the surface of the tube on the switch which received it, the switch itself being a vertical knife-edge extension of this surface, which passed through the vertically descending water at starting and stopping; and further, to prevent any minute drops of water going astray from the bursting of an occasional bubble in passing, a sheet brass hood was placed round the descending pipe directly the trial started.

The outside of the weighing tank is completely exposed to observation, and is perfectly tight. The valve in the bottom, being a 4-inch leather-faced screw-valve on a brass seat, is also tight, but for satisfaction it was arranged to place a clean tin dish under the valve before starting a trial, and only to remove it after the water was weighed, so that there should be absolutely no loss of water from any of these causes.

That there must be some loss of water by evaporation to the air as long as the temperature of the water, after leaving the condenser, was above that of the dew-point of the surrounding air, was certain. By using sufficient cooling water it would be possible to bring the temperature down to that of the dew-point; but it was found that this could not be done under all circumstances without a larger condenser, for which room was wanting, and, as long as the water lost by evaporation was the

same in both trials, all error would be eliminated in the difference of the large and small trials. After careful consideration, it was arranged that the condensing water should be adjusted so that the water in all trials entered the tank at a temperature as nearly as possible 85°; it being probable, as the surface exposed to the air was nearly the same in the large and small trials, if the differences in temperature between the air and the water were the same, the evaporation would be the same, or would at least differ by a constant amount. In order to test this, it was further arranged that, after the trials were finished, the centrifugal pump should be temporarily re-arranged so that it could be used to draw water out of the tank and force it round through the condenser and switch, and so back again into the tank at rates corresponding to those of the large and small trials, and at the same temperature (85°), the water in the tank being at this temperature, the arrangement of the pump being such that, when stopped, all the water in the pipes would run back again into the tank. This would practically insure the same loss of water by evaporation during one hour's pumping as during one hour's trials, and any difference (w_e) thus established between the large and small trials would then be treated as a standing correction on the difference of the heavy and light trials. This relative correction, taking W as the mean difference of water in the heavy and light trials, would be

$$\frac{w_e}{\mathrm{W}}.$$

The Standards of Measurement.

38. In these experiments the expressions obtained for the work done in heating the water and the heat generated are, respectively,

$$2\pi\mathrm{N}\,.\,\mathrm{RW}_i \quad \text{and} \quad \mathrm{SW}_\omega(\mathrm{T}_2{}^\circ - \mathrm{T}_1{}^\circ),$$

where R, W, T°, S are respectively length, weight, temperature, and capacity for heat.

Since these expressions both represent the same absolute quantity of energy, the difference in the numerical values of these expressions results only from the difference in the units in the two expressions. These units may be considered as the unit of work and the unit of heat respectively, as it is the inverse ratio of these units, measured in absolute quantities of energy, that is expressed by the ratio obtained from

$$\frac{2\pi\mathrm{NRW}_i}{\mathrm{SW}_\omega(\mathrm{T}_2 - \mathrm{T}_1)}.$$

But, as there are no actual standards either of work or heat with which quantities of work and heat can be respectively compared by a simple measurement, such comparisons can only be accomplished by the comparison of the several factors involved

in each of these expressions with the several absolute standards which exist for such factors.

These standards are the standards of mass, length, and force, on the one hand, and of mass, quality of matter, and temperature, on the other.

Thus, work being defined as the mean product of force multiplied by the distance, and the standard of force being the force of gravitation on the unit of mass wherever it occurs, the work is represented by $W.h$, where W expresses the number of units of mass, and h the number of units of length through which it has been raised. Taking (M) and (L) as expressing these units, the unit of work is expressed as (ML).

Again, the unit of heat is defined to be one n^{th} part of that quantity which is required to raise one unit of mass (M) of a standard substance (pure water) from one definite state of temperature to another definite state. And calling this interval θ, the unit of temperature is defined to be θ/n. And, taking S to express the ratio of the number of units of heat required to raise W_ω units of mass of matter from $T_1°$ to $T_2°$ compared with $W_\omega(T_2° - T_1°)$, the heat expressed by $SW_\omega(T_2 - T_1)$ is in units $\left(M\frac{\theta}{n}\right)$.

So that, from the physical equivalence of the absolute energy expressed in the respective forms, it appears that the unit of heat, as defined by $\left(M\frac{\theta}{n}\right)$, is equivalent to

$$\frac{2\pi NRW}{SW_\omega(T_2-T_1)} \text{ units of work as defined by (ML),}$$

or that the heat required to raise one unit of mass of pure water through the definite interval of temperature θ is equivalent to

$$n\frac{2\pi NRW}{SW_\omega(T_2-T_1)} \text{ units of work (ML).}$$

This is the definition of the mechanical equivalent of heat in Manchester, adopted by JOULE if $n = 1$, and θ is 1° Fahr. between 50 and 60, as determined on his thermometer. But, since the absolute kinetic value of the unit of force as here defined varies with the latitude and height of the place, while that of the unit of heat is constant, this mechanical equivalent varies from place to place with $1/g$, where g is the expression, in kinetic units, for the unit of force (M).

Thus, expressing the work in kinetic units, the unit of heat, as already defined, is equivalent to

$$g\frac{2\pi NRW}{SW_\omega(T_2-T_1)} = C,$$

where the dimensions of C are $(L^2T^{-2}n\theta^{-1})$.

Whence, since g has dimension (LT^{-2}),

$$\frac{2\pi NRW}{SW_\omega(T_2 - T_1)} = \frac{C}{g},$$

where the dimensions of C/g are $(Ln\theta^{-1})$.

The object in this research being to replace the standard of temperature, as defined by the scale on a particular thermometer, by the standard obtained from the states physically defined by melting ice and by water boiling under a standard pressure, θ is here defined to express this interval, and S is, in accordance with the definition already given, used to express the ratio which the heat required to raise unit mass over any interval, per degree of rise, bears to that required to raise pure water over the interval θ, per degree of rise.

The Standards Involved.

39. It appears from the dimensions of C/g, as obtained in the last article, that the only general standards to which reference need be made are those of length and temperature.

It is, however, to be noticed that the determination of the work and the heat involve the determination of separate masses, and that the units only disappear on the condition that they are equal.

The Measurement of Mass.

40. Since it was not necessary to refer the mass to a general standard, the weights used were only referred to a Board of Trade standard for convenience.

Thirteen of the 25 lb. weights used for loading the brake were adjusted to the Board of Trade weight, then carefully balanced against each other, till, balanced in groups of four in any arrangement, there was less than 0·01 lb. difference. Four of these weights were then taken as the standard.

The compound lever machine, which had two scales on the same lever, one notched to each 100 lbs. for the position of the large rider, the other with a flat scale for every 1 lb. for the position of the small rider, was taken to pieces and the knife edges re-ground and re-set (by MR. FOSTER) till consistent results were obtained to the one-hundredth of 1 lb. Another rider was also made to work on the same scale as the small rider, being adjusted to one-hundredth of the weight, so as to read 0·01 lb.

The scales were then carefully surveyed by the standard 100 lb. weight, the original small rider being adjusted till the difference between its extreme positions on the scale balanced the standard to $< 0{\cdot}01$ lb., and the corrections for each V-notch into which the feather on the large rider fitted ascertained by balancing the standard to a like degree of accuracy.

The dead load on the scales, including the empty tank, came to 340 lbs., about, and between this and 2200 lbs. the scales would weigh any quantity with the lever swinging to 0·01 lb.

The weights to which the scales had been adjusted were then exclusively used on the brake. Thus the brake was balanced by the same weights as were used as the standard in weighing the water, with a sensitiveness which gave the error less than one forty-thousandth part of the weight of water in the smallest trials, while the casual error, which would not exceed this in a single weighing, would be eliminated in the mean of a large number of weighings. Thus the relative limits of error in weighing would not exceed ·000025.

The Correction for the Weight of the Atmosphere.

41. The balances being made in air it is necessary to add the weight of air displaced in each case.

As the relative weights only are concerned, if D_a is the weight of a unit volume of air, D_w that of water, and D_i that of cast-iron, the weights in air of unit masses are :—

$$1 - D_a/D_w \quad . \quad . \quad . \quad . \quad \text{for water,}$$
$$1 - D_a/D_i \quad . \quad . \quad . \quad . \quad \text{for cast iron.}$$

The load on the brake is therefore subject to the correction expressed by the factor $(1 - D_a/D_i)$, while that of the water balanced against cast-iron weights, has the correction factor

$$\frac{1 - D_a/D_i}{1 - D_a/D_w},$$

and the relative correction for the actual weight of water, as against the load on the brake in air, is

$$1\Big/\left(1 - \frac{D_a}{D_w}\right) \text{ or approximately } 1 + \frac{D_a}{D_w},$$

for the temperature 67° Fahr., $D_a = 0{\cdot}0752$, $D_w = 624$.

Hence, the relative correction factor for the equivalent

$$(1 - 0{\cdot}001205).$$

The Correction for g in Latitude of Greenwich and 45°.

42. Since the latitude of Manchester is 53° 29′, Greenwich 51° 29′, the value of g being (" Mémoires sur le Pendule," ' Soc. Française de Physique ')

$$g_{45^\circ}(1 - 0{\cdot}00259 \cos 2\lambda) = g_{45^\circ}(1 + 0{\cdot}0007558) \text{ at Manchester,}$$

$$\quad\text{,,} \qquad \text{,,} \qquad = g_{45^\circ}(1 + 0{\cdot}0005814) \text{ at Greenwich,}$$

whence the correction factor is . . . . $(1 + 0{\cdot}0001746)$ for Greenwich,
and for 45° . . . . $(1 + 0{\cdot}0007558)$.

The Specific Heat of the Water.

The standard capacity for heat being that of distilled water, the obvious course would have been to have used distilled water in the trials, had this been practicable; but as it was apparent from the first that the quantity of water which would have to pass through the brakes during the trials would amount to some 20,000 gallons, or, say, 100 tons, all of which would have to be brought down to a temperature of 32° Fahr.; and that to do this, using distilled water, whether or not the water was used over again, the necessary appliance for producing, storing and cooling the water, were impracticable in the laboratory, the last 40° must be removed with ice, and this would require some 25 or 30 tons of ice. While using the town's water direct from the main, the average temperature, from February to June, would not exceed 45°, so that only 12° or 13° would have to be removed by ice, which would require from 7 to 10 tons, with no other appliances except the relatively small appliance for cooling.

The only practical course, therefore, was to use the town's water. And had it not been for the known purity of this, the research would never have been undertaken.

As affording definite assurance of the consistent purity of this water, as delivered in the college, Professor DIXON kindly undertook to furnish the mean results of the analyses which he makes periodically for the Manchester Corporation, of the water drawn from the supply in the college. These show that the impurities are almost negligible, taking 0·2 as the specific heat of the salts, the relative correction is $0{\cdot}8s$, where s is the relative weight of the salts.

The Effect of Air in the Water.

43. Even distilled water contains air unless special precautions are taken for its removal; so that any effect such air may have on the capacity for heat as measured would not have been avoided by using distilled water.

The direct effect of the same 0·00323 per cent. of air which water exposed to the atmosphere usually contains at normal temperatures, is so small as to be altogether

negligible, and it would seem to be an open question whether the standard condition of water, as regards capacity for heat, does not involve the inclusion of this air. But the indirect effect of such air on the heat necessary to raise water from normal temperatures to near the boiling-point, is by no means negligible.

It does not appear that any definite study has hitherto been made of this effect; but it is a matter of common observation that as water reaches a temperature some 40° Fahr. below the boiling-point, bubbles appear on the sides and bottom of the vessel, which gradually increase in size and rise to the surface, increasing rapidly in size as they rise. The bubbles are usually referred to as bubbles of gas or air. But, a moment's consideration will show that, although the air or gas is the immediate cause of the premature formation and subsequent expansion of the bubble, it is none the less certain that the space occupied by the bubble is filled with saturated steam at the temperature of the water, the function of the air being merely that of balancing the excess of pressure of the surrounding water over the pressure of the saturated steam.

It thus appears that every bubble so formed represents a quantity of heat which is the latent heat of the volume of the saturated steam in the bubble over and above the heat of the weight of water in this steam.

Thus, if bubbles of air exist in water at a temperature of 212° Fahr., the weight of air per lb. of water being a, 0·0000323, and p the pressure, in inches of mercury, of the water, then, since the pressure of the air is $p - 30$, and the volume of 1 lb. of air at 212° Fahr. under 30 inches of mercury is 16·9 cubic feet, the volume of air per lb. of water is

$$V = \frac{16{\cdot}9 \times 30}{p - 30} \times a,$$

or, if $p = 40$,

$$V = 50{\cdot}7 \times a.$$

This is the volume, in cubic feet, of saturated steam at 212°; whence, since the latent heat per cubic foot is 36·6 at 212°, the excess of heat will be per lb. of water

$$V \times 36{\cdot}6 = 1855 \times a,$$

and this, divided by 180°, gives a relative error

$$10{\cdot}31 \times a.$$

If $a = 0{\cdot}0000323$, the error is

$$0{\cdot}00033, \text{ or } 0{\cdot}033 \text{ per cent.}$$

The water, after being exposed to the atmosphere in the service reservoir, where it discharges any excess of air, enters the brake cold with this normal air, there it is

heated by work, under the pressure of the artificial atmosphere at pressure p, to maintain which it parts with some of the air, which, in passing out into the flexible pipe, carries out saturated steam, which is condensed by radiation from the pipe. The water, with the remainder of the air, is then carried by the centrifugal pressure into the outer chamber in the brake case under a pressure of about 50 inches of mercury. It then passes the automatic cock into the flexible pipe at 41 inches pressure, thence rising to the thermometer bulbs at 40 inches. In passing the automatic cock with a difference of pressure of 9 inches, the pressure will be further reduced, probably 9 inches below that in the pipe, so that any air that might have been retained would come out at that point, and expand further as it approached the thermometer bulb.

In the first instance, it was thought that a pressure of 5 feet of water would prevent the formation of bubbles, and the air gap in the pipe leading from the condenser was placed at this height above the thermometer. But many, and sometimes large, bubbles of air were observed passing up the thermometer chamber; and Mr. MOORBY observed that he could detect the passage of a large bubble by a fall in the thermometer before the bubble appeared in the glass chamber.

To prevent this, the air-gap was raised till it was 12 feet above the thermometer bulb; so that the error is limited to three ten-thousandths. Even so, as it is much larger than any of the errors of constant sign, it was important to try, by assimilating the conditions under which the water leaves the brake, to obtain experimental evidence which would narrow the limits.

It may appear at first sight as though these losses from the air in the water would, like the radiation, be eliminated in the difference of the large and small trials, but this is not so, since the quantity of heat so lost is proportional to the amount of water used, or it may be greater in the heavy trials.

The Standard of Length.

44. The measures of length that the research involves are—

(1.) The horizontal distance of the centres of gravity of the adjustable loads on the brake from the axis of the shaft.

(2.) The vertical heights of the barometer at which the boiling-points of the water were determined.

In order to secure a definite reference of these to the British standard, recourse was had to two carefully-preserved and independent measures derived from this standard.

(1.) A set of gauges by Sir JOSEPH WHITWORTH and Co., consisting of three steel bars, 9, 6, and 3 inches respectively, with parallel plane ends $\frac{3}{8}$ inch in diameter, adapted to a 20,000 of an inch measuring machine, which constitute the standards used in the engineering laboratory.

(2.) A brass bar by ELLIOTT and Co., 39 inches long, and graduated in inches, used as the standard in the physical department in Owens College.

From the Whitworth gauges, two steel bars, $\frac{3}{4}$ inch in diameter and 9 inches long, with parallel plane ends, were made by Mr. FOSTER, and compared with the 9-inch Whitworth bar by the measuring machine.

With these and the Whitworth gauges, placed end to end, an outside gauge consisting of two surfaced angle-plates on a surfaced cast-iron bed was set out, and then a steel bar $\frac{3}{4}$ inch in diameter with plane ends fitted to these. Careful comparison showed that this bar did not differ from the sum of the lengths of the gauges by $\frac{3}{10000}$ parts of an inch. This length was then carefully laid off by the surfaced angle plates on the surface plate, and was so compared with the scale of the Elliott brass bar, account being taken of the temperature, and found to agree within less than $\frac{3}{10000}$ of an inch.

The 30-inch bar so obtained was then taken as the standard both for the levers of the brake and the barometer, to be carefully preserved.

Lengths of the Levers.

45. The V-groove, in which the knife-edge of the carrier, by which the load on the brake was suspended, rested, was originally made at a distance of four feet from the axis of the shaft at ordinary temperatures, and, as whatever the error might be when the brakes were hot, it would be the same for all the trials, since the temperatures were the same, it was decided to take this as the length of the levers in estimating the loads during the progress of the research, and to treat whatever error there might be as a standing correction on the final results. Such correction to be obtained by laying off four feet less the radius of the shaft from the carefully squared end of a steel plate 3 inches broad, $\frac{3}{16}$ inch thick, then placing this, flat, in a vertical plane perpendicular to the shaft, with its edge horizontal, as near as practicable to the knife-edge groove with the squared end touching the shaft. Then by means of a theodolite, set so that its line of collimation was in a vertical plane parallel to the axis of the shaft, and intersecting the vertical line on the plate, to observe the distance of the groove from the line on the plate, while the brake was running under the same conditions of temperature, and load as in the trials; but with the carrier temporarily displaced further along the shaft, so as to leave the bottom of the V-groove visible through the theodolite and in this way to obtain the actual distance of the groove from the axis of the shaft as affected by the expansion of the brake and any displacement of the bearing on the shaft which might result from the running.

By using a scale divided to the one-hundredth of an inch, and taking several readings, this could be determined to a thousandth of an inch, so that the limits of accuracy would be

$$\pm 0 \cdot 00002.$$

The Standard of Temperature.

46. As the most general standard is the difference between the two physically fixed points of temperature corresponding to the temperature of ice melting under the pressure of the atmosphere and that of water boiling under a pressure corresponding to 760 millims. of ice-cold mercury in the latitude of 45°, taking account of the variation of g, the standard in Manchester is the interval between melting ice and water boiling under a pressure of $760 \times 1{\cdot}0001721$ millim. of ice-cold mercury, which corresponds to 29·899 inches. And this interval divided by 180 is one degree Fahr.

According to REGNAULT's tables a divergence of one thousandth of an inch from the boiling point would correspond to an error of 0·0017° Fahr., and this would be less than the one-hundred-thousandth part of 180°.

In order to obtain this degree of accuracy in comparing the pressure of the vapour of pure water, in which thermometers could be placed, with the height of mercury over a range of two or three degrees above, and two or three below the point, at almost any time, irrespective of what might be the actual pressure of the atmosphere, it was necessary that the barometer, or pressure gauge, while in free communication with the vapour chamber should be shut off from the atmosphere, and at the same time so far removed that the temperature of the mercury should not be affected by the heat from the gas or boiling water. And, further, although in direct communication with the vapour, this must be such that no moisture could reach the mercury; and, such as involved no current in the passages which might affect the relative pressures, as would result by the interposition of a condensing vessel.

It was also necessary that the arrangements for reading the vertical distances between the upper and lower surfaces of the mercury should not only give absolute differences of height, but also that they should afford ready means of at any time determining the presence of vapour or gas, other than that of mercury, in the upper limb of the barometer.

The Barometer.

47. To meet these requirements, the barometer shown (Plate 8) was designed. The vessel which holds the mercury consists of a bottle-shaped casting of iron, 3 inches in diameter. Through a stuffing-box in the neck of this, the stem of the barometer tube passes. To admit of reading the level of the surface of the mercury in the bottle, two parallel plate-glass windows are arranged, $\frac{3}{4}$ inch diameter, having their axis $\frac{3}{4}$ inch from the axis of the bottle. These are sunk into the casting so as to leave the outer cylindrical surface of the bottle clear, the joints between the glass and the cast-iron being faced and made tight with a trace of beeswax, the other openings into the bottle being one for the admission and abstraction of mercury, fitted with a screwed valve, and one for the admission of air, with a mouthpiece for the attachment of a tube from the vapour chamber.

The glass stem of the barometer is drawn down into a neck towards the lower end, and this is bent through 180° so as to bring the mouth upwards, and thus admit of its introduction into the mercury in the bottle without letting in air. This bend has to be passed through the stuffing-box, then the tube is secured by screwing the gland on to the beeswax stopping. A brass guard tube is then screwed into the neck, to support the glass tube, to a height of 24 inches from the mercury in the vessel.

For reading the height of the lower limb, a cylindrical brass curtain, with a conical contraction on the top, the aperture in which is threaded internally at twenty threads to an inch to correspond to the screw on the outside of the neck of the bottle, is screwed on to this neck, the lip or bottom of the curtain being truly turned so that, when screwed down to the level of the mercury, it cuts off the light through the windows from a white sheet behind.

To the top of the brass casting, which forms the curtain, a brass cylindrical tube is rigidly attached coaxial with the curtain which fits over the brass guard round the barometer tube, this extends to a height of 26 inches from the lower lip, the internal diameter for the last inch being a little smaller and internally screwed at twenty threads to an inch. Into this is screwed a brass tube, externally screwed throughout its length, about 6 inches long, with parallel opposite slots $\frac{1}{8}$ inch wide extending to within an inch at either end, to form windows through which to see the light over the upper limb of the mercury. And on to the upper portion of this tube there is screwed a long cap, capable of screwing down to the bottom of the slot. The lower lip of this cap forms the curtain which cuts off the light when the lip is level with the upper limb of the mercury.

By this arrangement the variation of the distance between the lips of the lower and the upper curtains depends only on the change in their relative angular positions. For, since the slotted tube has a uniform thread, it can be turned, screwing into the lower curtain and out of the upper, both of which remain unmoved. Thus the position of the windows may be fixed, while the curtains are moved. So that for reading the distances it is only necessary to measure the relative angle.

This angle is measured by dividing the circumference of the cap just above the lip into five equal divisions, from 0 to 5, and these again into ten, then a turn through one of the smaller divisions means an alteration in the distance of one-fiftieth of one-twentieth of an inch. As this angle is measured relatively to the lower curtain, a vertical brass scale, divided to tenths and twentieths of an inch, is fixed externally to the top of the extension of the lower curtain, extending vertically just outside the graduated limb of the upper curtain, and thus serves for reading the angular distance of the index mark on the limb of the upper curtain on any particular thread and the number of threads from the index on the scale.

The Adjustment of the Indices on the Barometer.

48. The lower curtain, together with the slotted tube and cap, is unscrewed from the neck of the cast-iron bottle and lifted off over the tube. Then the 30-inch standard bar is set on end upright on a surface plate, and the lower curtain, &c., are lowered over the bar until the lower lip of the curtain rests on the surface plate, and the top of the bar is 30 inches from this lip. The cap is then screwed down until light is seen over the top of the bar through the slot just cut off. Then a vertical line, drawn on the cap just above the lip, at the edge of the scale is the index on the cap, and a horizontal line, drawn on the scale level with the lip of the cap, is the index point on the scale. And, when these two lines are brought into this position, the distance between the lips will equal the length of the bar.

In order to check this the curtain is raised, and two thin pieces of chemical paper are placed on the surface plate, one on each side of the bar, so as to leave a space between the paper and the bar. Then the curtain is replaced so that it rests on the paper, and light can be seen through the interval between the paper and the bar. Then light should be seen to an equal extent over the bar, and by screwing down the cap till the light disappears, the thickness of the paper will be measured by the angle turned through.

The construction of this barometer, the first of its kind, was undertaken by Mr. FOSTER, who has produced a very beautiful instrument by which direct reading can be taken to the ten-thousandth of an inch. The mercury having been re-evaporated for the purpose in an apparatus belonging to Dr. SCHUSTER by his assistant, Mr. S. STANTON.

This barometer could be used as a pressure gauge for pressure up to 34 inches and down to 26 inches, and by connecting the mouthpiece with a receiver in connection with a mercury or water syphon gauge, with the other limb open to the atmosphere, the differences of reading of the barometer for different pressures in the receiver can be readily compared with the corresponding differences in the syphon gauge, and by such comparisons, taken at intervals till the mercury reaches the closing in of the tube, a test is obtained as to the absence of anything but mercury vapour above the mercury.

When the barometer is in connection with the vapour chamber in which the thermometer is immersed, the passage of moisture back into the barometer is prevented by connecting the tube by a branch with an air receiver, in which the pressure is maintained higher than that in the vapour chamber; the branch pipe communicating with the chamber through a piece of quarter-inch glass pipe, 3 inches long, plugged as tightly as possible throughout its length with cotton wool, through which the air has to pass from the receiver into the vapour chamber. In this way an indefinitely slow current of clean dry air can be maintained into the passage from the vapour chamber to the valve which controls the exit of the steam into the

atmosphere, so that the air does not enter the vapour chamber in which the thermometers are, but directly passes out with the overflow steam.

There is necessarily some resistance to the air passing along the pipe to the vapour chamber, but this could easily be tested by removing the pipe from the vapour chamber, and leaving it open to the atmosphere, so that the barometer would adjust itself to that of the atmosphere, plus the pressure due to the resistance of the current in the pipe ; then, stopping the current by closing the branch pipe, and reading again, the difference would give the pressure due to the current. With the plug as described this was so small as to be negligible, even when the pressure in the receiver was two atmospheres. As during the testing of the thermometers the pressure in the vapour chamber was generally greater than that of the atmosphere, in order to maintain this steady, a governor on the gas burner was necessary, as well as an accurately adjustable exit valve.

With these appliances the scale of high temperature thermometer could be tested at intervals, over a sufficient interval on each side of the boiling point (212° Fahr.), the corrections for surface tension, temperature, and gravitation being applied to within the thousandth of an inch of mercury.

This gives the limits of error $\pm$ 0·00001.

Correction of the Low Temperature Thermometer.

49. The correction on the thermometer for 32° would be at any time obtained in the usual way by immersing the thermometer vertically in a bath of soft snow, but as there was no ready means, as with the scale about 212°, of testing the scale at 32°, while this would be used for one or two degrees, this correction could only be made by comparison with a thermometer already corrected with the air thermometer, which comparison Dr. SCHUSTER allowed to be made in the physical department.

Corrections of the Thermometers for Pressure.

50. The pressures in the thermometer chambers of the brake being both some 10 or 15 inches of mercury above that of the atmosphere, it would be necessary to determine the corrections on each of the thermometers under the pressures and temperatures at which they had to work.

Thus, if e_1, e_2 are the corrections per unit of pressure in the initial and final thermometers, the correction for the heat is $(e_1p_1 - e_2p_2)$.

The Range of Temperature over which the Specific Heat would be Measured.

51. The temperature of the effluent water from the brake can be regulated either up or down to any required extent, and although there would necessarily be some

divergence from the boiling-point, with care and experience it would be possible to bring the mean result in a number of trials within a close approximation of 212° Fahr.

On the other hand, there has been no means provided of regulating the temperature of the water entering the brake. This is determined by the rate at which the water passes through the iced coil and the temperature at which it entered, as determined by the temperature in the town's mains, which varies from 38° in the winter to 55° in the summer. Thus the temperature in the light trials would be from half to a degree above 32°, and that of the heavy trials from a degree to two degrees.

In calculating the heat of each trial the actual difference with the correction for the thermometers is taken, but if, as is shown by previous investigations by REGNAULT and others, the specific heat at and near 32° is less than the mean specific heat between 32° and 212° by something like 0·5 per cent., there would be errors in taking the results so obtained as the mean specific heat between 32° and 212°.

Owing to the extreme difficulty of determining the specific heat over a very short range of temperature to such high degrees of accuracy as ·01 per cent., the experimental evidence as to the exact value of the specific heat within a few degrees of 32° is but vaguely surmised from the general fall of the specific heat with the temperature.

The law of the thermal capacity of water between 0° C. and $t°$, as deduced by REGNAULT from his experiments, is avowedly vague as to the lower temperatures. It shows no singular point at the maximum density, as would be expected; and RANKIN deduced another law from these experiments, making the minimum specific heat coincide with the point of maximum density. Also other experimenters have obtained higher specific heats near 32° than are given by REGNAULT'S formula. It would seem probable, therefore, that the difference between the specific heat at 32° and the mean between 32° and 212°, as given by REGNAULT'S formula, is too large.

In that case, the correction obtained by this formula in order to reduce the specific heat between the observed temperature in the trials to that between the standard points, would probably be too large, and thus afford an outside limit of error.

Thus, putting s for the mean specific heat between 32° and 212°, $s\,(1 + \mathrm{X})$ for the specific heat between $\mathrm{T}_1°$ and 212°, when $\mathrm{T}_1°$ is small compared with 180°, and, by REGNAULT, taking $s\,(1 - 0{\cdot}005)$ for the specific heat at $\mathrm{T}_1°$, then the total heat from $\mathrm{T}_1°$ to 212° is

$$\begin{aligned} s\,(1 + \mathrm{X})\,(212 - \mathrm{T}_1°) &= s\,\{180 - (\mathrm{T}_1° - 32)\,(1 - 0{\cdot}005)\} \\ &= s\,(212 - \mathrm{T}_1°)\left(1 - \frac{\mathrm{T}_1° - 32}{212 - \mathrm{T}_1°} \times 0{\cdot}005\right), \end{aligned}$$

or, neglecting $(\mathrm{T}_1 - 32)^2$,

$$\mathrm{X} = 0{\cdot}005\,\frac{\mathrm{T}_1° - 32}{180} = 0{\cdot}000028\,(\mathrm{T}_1° - 32).$$

Thus, taking the mean capacity of water between the temperatures of 32° and 212° as the standard capacity, the mean specific heat between $T_1°$ and 212° would be

$$1 + X = 1 + 0{\cdot}000028\,(T_1° - 32)\,;$$

and, if $T_1°$ is the mean initial temperature of the water of any number of trials, $1 + X$ is the mean specific heat of the water in all the trials. The mean specific heat of the difference of two trials would be $1 + X$; this appears as follows:—

Suppose $1 + X_1$ to be the mean specific heat for a set of heavy trials, and W_1 the mean weight of water, and $(1 + X_2)$ to be mean specific heat of a corresponding set of light trials, and W_2 the mean weight of water, $T_1°$, $T_2°$ being respectively the initial temperatures of W_1 and W_2, the difference of the total heats would be

$$(1 + X_1)\,(212 - T_1°)\,W_1 - (1 + X_2)\,(212 - T_2)\,W_2,$$

and the mean specific heat would be approximately

$$\frac{(212 - T_1°)\,W_1 - (212 - T_2°)\,W_2 + 180\,(X_1W_1 - X_2W_2)}{(212 - T_1)\,W_1 - (212 - T_2)\,W_2} = 1 + \frac{180\,(X_1W_1 - X_2W_2)}{180\,(W_1 - W_2)}\,;$$

and, as in the heavy and light trials $W_1 = 2W_2$ approximately, the mean specific heat by REGNAULT's formula would be

$$1 + 2X_1 - X_2 = 1 + 0{\cdot}000028\,[2\,(T_1 - 32) - (T_2 - 32)].$$

This result is obtained by merely summing the trials, but counting the water in the light trials as negative,

$$X = {\cdot}000028\,\Sigma\left\{\frac{W\,(T_1 - 32)}{\Sigma\,(W)}\right\}.$$

The Gradual Rising of the Indices of the Thermometer.

52. Where, as is generally the case, the indices of the thermometers are gradually rising, if they are used between the intervals at which they are corrected, the last observed correction being applied, there will be an error which will be negative, and of magnitude equal to the rate of rise during the interval multiplied by the interval. Thus, if the trials are uniformly distributed between the intervals of correction, the correction would be $0{\cdot}5a$, where a is the observed rise in the interval, hence the relative correction on the equivalent, taking $\bar{a}_1$ and $\bar{a}_2$, as the mean rises between the intervals of correction of the initial and final thermometers, would be

$$\frac{0{\cdot}5}{180}\cdot(\bar{a}_2 - \bar{a}_1).$$

The Work done by Gravity on the Water.

53. The difference of pressure on the bulbs of the initial and final thermometers which are at the same level, expressed in feet of water, is the work done by gravity per lb. of water. If p_1 and p_2 express these pressures in inches of mercury, the work done by gravity is

$$1 \cdot 14 \, (p_1 - p_2),$$

which gives as the relative correction for the equivalent, approximately,

$$+ \, 0 \cdot 000008 \, \Sigma [\mathrm{W} \, (p_1 - p_2)] / \Sigma \, (\mathrm{W}).$$

The Work absorbed in Wearing the Metal of the Bushes and Shaft.

[54. During the six years the brake had been in use, before the trial commenced, the shaft and bushes were occasionally lubricated with oil, chiefly to prevent oxidation of the shaft when standing, and, up to the commencement of the trials, there was hardly any appreciable sign of wear. After the closing of the bushes by the stuffing-box and cap, when the use of oil was purposely discontinued, there was no means of observing the wear of the metal as long as the brake worked satisfactorily, as it did during all the trials. But when, after the completion of the trials, the stuffing-box and cap were removed, in order to return to the original manner of working, the excess of leaking through the bushes showed that there had been considerable wear.

At that time it did not occur to me that the proportion of this wear, which took place during the actual running of the trials, would represent a certain amount of work absorbed in disintegrating the metal, or a certain amount of heat developed by the oxidation of the metal, and no attempt was then made to form a definite estimate of the amount of metal which had disappeared. As, however, the worn metal was replaced by a coating of white metal, the thickness of this (less than $\frac{1}{32}$nd of an inch) and the extent of surface (less than 124 square inches) subsequently showed that it could not be more than 1 lb.

This was after it occurred to me that however small might be the effect of this wear, since it was definitely observed to have taken place during the twelve months when the bushes were closed for the purpose of the trials, it was desirable, in order to complete the research, that some outside estimate should be obtained of the limits to its possible effect, whether from disintegration or from oxidation.

In as far as the loss of metal was due to the abrasion of the clean metal surfaces, it would be proportional to the number of revolutions, while in as far as it was owing to the oxidation of the metal surfaces, left bright after each run, it would be probably proportional to the number of runs.

The number of revolutions with the bushes closed, counting ordinary work as well as the trials, is found from the records to be less than $300 \times 60 \times 360$, and the number of runs to be 80, the mean time being 4·5 hours. The revolutions during any one of the accepted trials were 300×60. And the trials were made in threes, so that the coefficient for oxidation would be $\frac{1}{240}$.

Hence, the metal worn by abrasion in a single trial would be less than $\frac{1}{360}$th of 1 lb. = 0·0028 lb., and the metal oxidised in one trial less than $\frac{1}{240}$th = 0·004 lb. So far the estimate is fairly definite, but, in order for its completion, it is necessary to arrive at some conclusion as to the work absorbed in disintegrating the metal, and of the heat developed by its oxidation.

There does not seem to be any reason why there should be more oxidation of the bright surfaces in a light trial than in a heavy trial, so that there would have been no error from this cause in their difference.

As regards the abrasion and the oxidation of the abraded metal, there would be a difference, as the weight on the shaft in a heavy trial is 1·23 of the weight in a light trial. Thus the differences of abrasion would have been

$$0{\cdot}0006 \text{ lb.}$$

The work necessary to produce a state of disintegration, such as exists in the vapour of the metal, would be the total heat of vaporization, less the kinetic energy and work $-[\kappa_v/(T-32)+PV]$, and, although the heat of vaporization of the metal is not known, it would seem that it cannot greatly exceed, when subject to the deductions mentioned, the heat of vaporization of ice subjected to like deductions (1,000,000 ft.-lbs.).

Assuming this, since the difference in the work of two trials is about 70,000,000 ft.-lbs., the correction would be

$$-\,0{\cdot}00001,$$

which, considering that the disintegration would be very imperfect, may be taken as an outside limit, while the effect may have been even reversed by the oxidation of the degraded metal.—Nov. 9, 1897.]

Accidents.

55. In contemplating such an extensive and complex research, the result of which depends on the mean of a number of experiments, it was impossible to overlook the question as to how such accidents, as would probably occur, should be dealt with.

It was clear that, whatever the rule might be, it must be definite and rigorously applied.

Two other things were also clear, that, as in surveying, accidents might occur, say

in reading the counter or the scales, which would only be apparent from the reduction of the results after the trial was finished. Also, that in these experiments there would be no such rigorous check on the results as in surveying; so that, without danger of sorting the results, anomalous results, the cause of which was not noted during the trial, could only be rejected when the results themselves contained evidence of the cause of the anomaly, say an abnormal difference between the mean speeds by the counter and the speed gauge.

It was therefore, from the first, decided to reject all trials in which there was definite evidence either during the trial or in the results, of uncertainty to which no definite limits could be assigned, in any one of the measurements, without regard for the apparent consistency of the results, and in the same way to retain all other trials.

56. The following table contains a summary of all those circumstances on which the accuracy of the result of the investigation depends, together with references to the several Articles in which they have been discussed. In line with each circumstance is placed the formula for the relative correction in the equivalent, necessary in consequence of the observed deviation from the conditions of equality between the heavy and light trials. In the same line with each circumstance are also given, to the millionth part, the limits of relative error as deduced in the corresponding Articles.

TABLE of Corrections (relative) and Limits of Error (relative) for all circumstances affecting the Accuracy of the Results.

Reference No.	References to Articles.	Circumstances affecting the accuracy of the results.	Formulæ for the relative corrections.	Limits of relative errors.	
1	6	Terminal differences in the moments of momentum of the water in the brake	0·00000	+0·000000	−0·000000
2	8	Cyclic-fluctuations in the speed of the engines	0·00000	0·000025	−0·000025
3	9	Work done on the water by end-play in the shaft	0·00000	0·000017	0·000000
4	9	,, ,, dash-pot	0·00000	0·000000	0·000000
5	9	Effect of the automatic-gear on the balance of the brake	0·00000	0·000016	0·000016
6	10	Imperfect elimination of the error of balance	0·00000	0·000000	0·000000
7	29	,, ,, ,, heat conducted by the shaft	$-C\Sigma(T_B^\circ - T_{SB}^\circ)/\Sigma(H)$	0·000000	0·000000
8	32	,, ,, ,, ,, radiated from the brake	$-R\Sigma(T_2^\circ - T_a^\circ)/\Sigma(H)$	0·000010 (?)	0·000010 (?)
9	34	The engagement of the counter	0·000013	0·000000	0·000000
10	31	Terminal differences in speed and temperature	$-\Sigma\{(B+w_i)T^\circ_i-(B+w_f)T^\circ_f\}/\Sigma(H)$	0·000000	0·000000
11	35	Leakage of the stuffing-box	$-\Sigma w_{SB}(T_{SB}^\circ - T^\circ_1)/\Sigma H$	0·000000	0·000025
12	36	,, at the automatic valve	0·00000	0·000000	0·000000
13	37	Imperfect elimination of the water lost by evaporation	0·00000	0·000000	0·000000
14	40	Limits of accuracy in weighing the water	0·00000	0·000025	0·000025
15	41	Weight of the atmosphere	−0·001205	0·0000000	0·000000
16	42	Correction for gravity at the latitude of Greenwich	0·000172	0·0000000	0·000000
17	42	,, ,, ,, ,, 45°	0·000745	0·0000000	0·000000
18	43	Salts dissolved in the water	$+0·8s$	0·0000000	0·000000
19	43	Air ,, ,, ,,	$-10·3/a$	0·000100 (?)	0·000100 (?)
20	45	Length of the lever	?	0·000020	0·000020
21	50, 46	Effect of pressure on the thermometers, limits of error	$-\Sigma\{W(e_1p_1 - e_2p_2)\}/\Sigma(W) \times 180$	0·000010	0·000010
22	52	Rise of the standard readings of the thermometers in the intervals of correction	+ 0·5 × (the difference of rise / the number intervals) / 180	0·000000	0·000000
23	51	Differences between the initial temperature of water and freezing	$-0·000028 . \Sigma\{W(T_1^\circ - 32^\circ)\}/\Sigma(W)$	0·000000	0·000000
24	53	Work done by gravitation on the water	$-0·000008\,\Sigma\{W(p_1 - p_2)\}/\Sigma(W)$	0·000000	0·000000
25	54	Work absorbed in wear of the metal	0·00000	0·000010	0·000010
			Summary of limits of error	+0·000233	−0 000241

The quantities included under the signs Σ (   ) are to be taken positive for the heavy trials and negative for the light trials.

Significance of the Symbols in the Formulæ.—C, the constant for conduction obtained from the trials; R, constant for radiation, as defined in trials, with the lagged and naked brake; s and a, are weights of *salts* and *air* in unit weight of water; p_1 p_2, pressures in inches of mercury in the initial and final thermometer chambers; e_1 e_2, are the corrections, per inch of mercury, pressure on the initial and final thermometers; W, weight of water used in a trial; w_{SB}, weight lost at the stuffing-box; T°, temperature Fahr.; T_{SB}°, stuffing-box; T_B°, bearing; T_a°, air; T_i, temperature at beginning; T_f, at the end; w_i, water in brake at beginning; H, the heat generated during the trial; B, capacity for heat of the metal in the brake.

PART II.

On an Experimental Determination of the Mechanical Equivalent of the Mean Specific Heat of Water between 32° and 212° Fahr., made in the Whitworth Engineering Laboratory, Owens College, on Professor Osborne Reynolds' method.—By William Henry Moorby, M.Sc.

1. In view of the frequent and extremely careful and accurate determinations of the value of the mechanical equivalent of heat which have been made of late years by different experimenters using different methods the present series of experiments may on first thoughts seem superfluous. There did, however, seem to be sufficient disagreement between the results previously published—more particularly between values of the equivalent, as derived from the direct methods described by Joule, Rowland, and Miculescu, and the indirect electrical methods of Griffiths, and Gannon, and Schuster, to warrant a new investigation into the value of this important constant, if the proposed new method of working should carry with it advantages not available in previous investigations. I was accordingly very glad to fall in with the wishes of Professor Reynolds that I should undertake a research bearing on this point on lines which he suggested to me in July, 1894.

2. In Part I., par. 3, a full description is given of the apparatus whose existence in the Whitworth Engineering Laboratory led up directly to the institution of this research into the value of the mechanical equivalent of heat.

The advantages which the proposed method offered were briefly :—

(1.) The possibility of obtaining a result which in no way depended for its accuracy on the value of the scale divisions of the thermometers used in the measurements of temperature (Part I., par. 11).

This was done by supplying a stream of water to the brake at a temperature of 32° Fahr., and there raising its temperature to 212° Fahr. before admitting it to the discharge pipe where its temperature was again taken.

(2.) A means of eliminating from the result all losses of heat due to radiation and conduction from the calorimeter employed (Part I., par. 32). The manner in which this elimination was accomplished is indicated below.

Let U and u represent the quantities of work done in two trials which differed only in the moment of resistance offered by the brake—the number of revolutions of the engine shaft and the duration of the trials being the same in each case.

Also let H′ and h' be the apparent quantities of heat generated in the brake in these trials. These quantities will be less than the true equivalents of the works U and u by quantities which represent the losses of heat from the brake by conduc-

tion, radiation, &c. These losses were made as nearly as possible equal by keeping the temperatures of the brake and its supports and surroundings at the same levels in the two trials.

Then the quantity of work $(U - u)$ should be exactly equivalent to the quantity of heat $(H' - h')$, and by dividing the first of these by the second, a value of the constant required is obtained.

The power available for the purposes of the investigation enabled me to deal with quantities approaching the following values in trials of one hour's duration :---

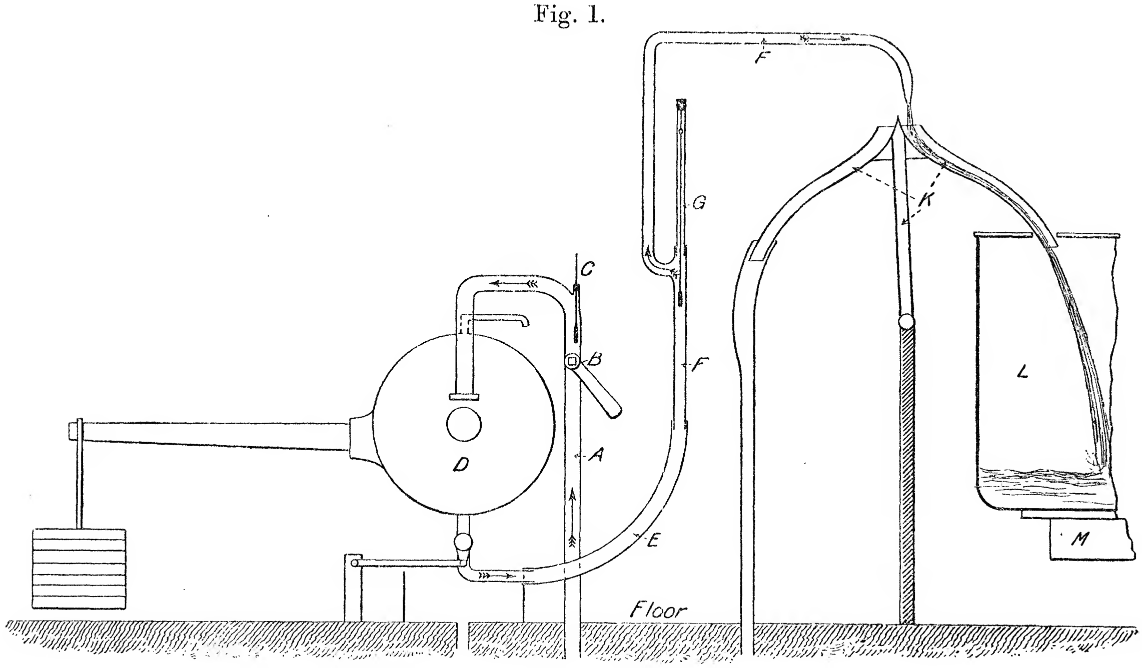
Revolutions, 18,000.
Total work done, 135,000,000 ft.-lbs.
Total weight of water raised 180° Fahr. = 960 lbs.
Total apparent heat generated = 170,000 B.T.U.

In quantities so large as these some of the small errors inevitable to all physical experiments became quite or nearly negligible.

Preliminary Apparatus and Trials.

3. It will, perhaps, be sufficient to indicate the general arrangement of the apparatus as first set up. This is illustrated in the annexed sketch. The water was

Fig. 1.



Preliminary Apparatus. Course of water shown by arrows.

supplied from the mains through the iron stand-pipe, A, and the regulating cock, B. Before it entered the brake its temperature was measured by means of the

thermometer, C, inserted through a cork in the stand-pipe, the part of the stem on which readings were taken being exposed to the atmosphere. After being discharged from the brake, D, the water entered a flexible rubber pipe, E, bent through an angle of 90°, which connected a horizontal nipple at the bottom of the brake with a vertical one forming the lower end of a fixed line of copper piping, F. The temperature of discharge of the water was indicated by the thermometer, G, which was enclosed in a glass tube opening through a stuffing-box into the discharge pipe, the whole length of the stem being therefore kept at the temperature of discharge. On leaving the copper discharge pipe the water was directed at will by the two-way tipping switch, K, either to the left to waste or to the right into the tank, L, standing on the platform of the weighing machine, M.

A series of trials were made with this apparatus, the water being raised through varying intervals of temperature between 35° Fahr. and 100° Fahr. For obvious reasons the results were not satisfactory, and are therefore not published. Experience was gained, however, which helped very materially in the design of the final apparatus.

Common thermometers were used, and calibration errors on the comparatively small range of temperature through which the water was raised were of sufficient importance to vitiate all results. Again, the exposure of the stem of the thermometer, C, was a weak spot in the apparatus. I was much troubled also with leakage of water from the two bushed bearings of the brake.

In so far as could be judged, the bent rubber pipe, E, was found to be a satisfactory connection between the brake and the copper discharge pipe, and this has been retained in the subsequent apparatus.

Details of the Constituent Parts of the Final Apparatus.

Artificial Atmosphere.—(Part I., par. 23.)

4. To prevent loss of water by evaporation at the centres of the vortices formed in the brake, the ports in the vanes of the outer casing were connected through a flexible rubber tube some 4 feet long, with an artificial atmosphere formed in a tin receiver, the pressure in which was maintained by means of a cycle tyre inflator at about 9 inches of mercury, as measured on a U-gauge. The shape of this vessel is made clear in the sketch (Part I., fig. 8). The ends were made conical for greater strength. The receiver was also provided with an air valve, with which to relieve the pressure when too high, and a cock, with which water accidentally lodging inside could be drained away.

The Ice Cooler.—(Part I., par. 19.)

5. Some preliminary experiments indicated that a length of about 200 feet of $\frac{3}{8}$-inch diameter lead piping would, when immersed in a mixture of ice and water, be sufficient to cool a stream of some 16 lbs. of water per minute very nearly to 32° Fahr.

The ice cooler was accordingly made as follows : A wooden box, 4′ 0″ × 2′ 3″ × 2′ 0″, and lined inside with waxed cloth, was fitted with a horizontal wooden shelf about 2 feet 6 inches long, and on this was laid a flat oval coil of $\frac{3}{8}$-inch composition piping nearly 200 feet in length, the left-hand end of the coil and shelf stopping short at a distance of 1 foot from the end of the box, the right-hand end of the coil reaching the end of the box, but the shelf stopping some 6 inches short of that point. The coil was about 5 inches diameter, vertically, and over it were placed the wooden guide plates shown (Part I., fig. 7). An 8-inch diameter paddle, having 6 wooden floats, was placed about the middle of the box, at a height just sufficient to ensure the lower edges of the floats clearing the coil of pipe below it. A galvanized iron wire netting, extending from the shelf upwards to the top, separated the well at the left-hand end of the box from the compartment to the right containing the coil and paddle.

When working, the well and space beneath the shelf contained broken ice, well rammed in ; while the level of the water was automatically kept at about 3 inches above the top of the coil. The paddle, driven by a cord from the line shafting in the engine-room, revolved in the direction shown by the arrow, and caused a circulation of water up through the ice in the well, and then horizontally through the coil and back to the ice under the shelf.

Circulating Pump.—(Part I., par. 20.)

6. In order to supply sufficient water to the brake against the resistance offered by the 200 feet of pipe in the cooler and the augmented pressure in the brake itself, it was necessary to use a circulating pump. This was a small MATHER-REYNOLDS centrifugal pump with four $1\frac{1}{2}$-inch wheels, driven by a turbine available for this purpose in the engine-room. This pump was capable of supplying 16 lbs. of water per minute, against a pressure of 25 lbs. per square inch at the supply valve.

Some difficulty was encountered in the summer of 1896 with this combination, because the excessive demand for condensing water for the engine hardly left sufficient flow in the falling hydraulic main to work the turbine at the requisite speed to maintain the above pressure.

On the whole, however, the combination was exceedingly efficient, and with a graduated supply valve afforded a very delicate means of regulating the flow of water into the brake.

Water-tight Joints between the Brake and the Engine Shaft.

7. In (Part I., par. 24–29) the necessity of obtaining control over the leakage of water at the bearings of the brake, and the methods by which this was accomplished, are fully discussed. The bearing on the up-shaft end of the brake was provided with a stuffing-box, while the shaft end was covered with a cap. The annexed sketches show the general design of the stuffing-box and cap :—

A—The engine crank shaft.

B—The outer skin of the brake.

C—Conical brass bushes screwed into the outer skin of the brake.

D—Lock nuts on these bushes.

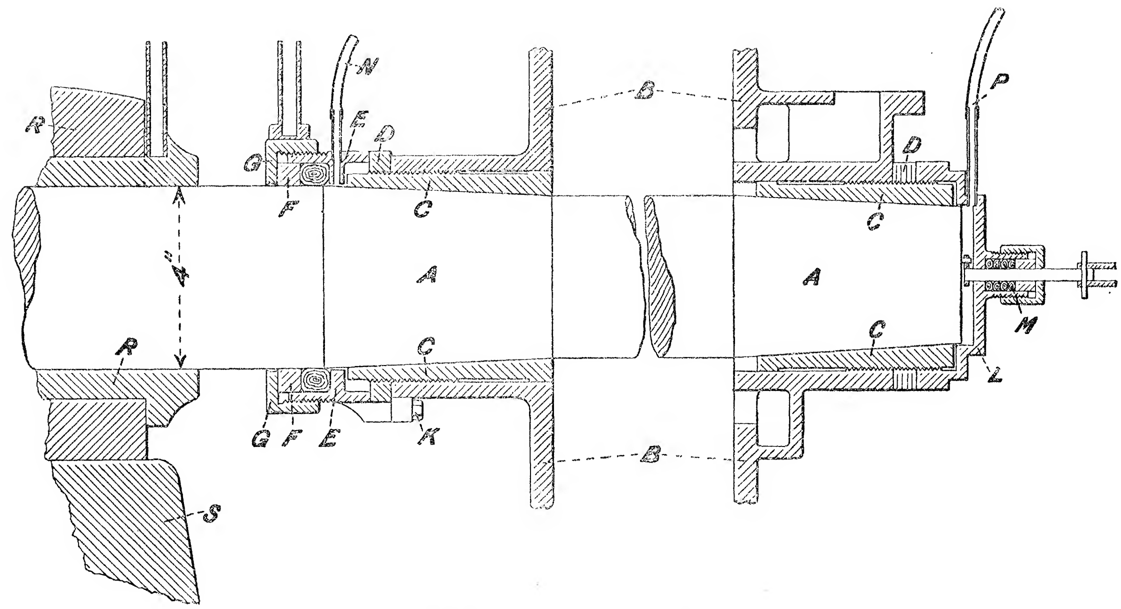
E, F, and G—Stuffing-box, ring and cover.

K—Set screws fastening stuffing-box to the lock nut.

L—Cap covering the end of the shaft.

M—Small spindle driven by a pin on the end of the engine shaft, passing through a stuffing-box on the cap, and required to drive the revolution counter.

Fig. 2.



Joints between brake and shaft.

The cap completely stopped all leakage from the bearing to which it was fixed, and, when the stuffing-box had worked for a short time, only a few drops of water escaped from the up-shaft bearing.

The brass bush bearings needed lubricating, and this was accomplished by supplying a small stream of water to each bearing through the pipes N and P, each provided with a regulating cock. This water came from the supply pipe between the ice

cooler and the regulating valve controlling the main supply to the brake. It was consequently under considerable pressure and at a temperature very little over 32° Fahr. The water thus supplied had, of course, to enter the brake, and the amount supplied afforded a very convenient means of controlling the temperatures of the bearings.

At a distance of $2\frac{3}{4}$ inches from the cap of the stuffing-box was the end of one of the main bearings, R, carried on the cast-iron pedestal, S.

It was important that I should have some control over the loss of heat by conduction along this length of shaft. Accordingly, two pieces of brass pipe were soldered on to the cap of the stuffing-box, while two others were screwed, the one in the upper and the other into the lower brass forming the main bearing. Thermometers were placed inside the tube affixed to the stuffing-box cap, which happened to be uppermost at the time, and into the two pipes screwed into the main bearing. It was then assumed that the loss of heat along the shaft would vary with the difference of temperature between the stuffing-box cap and the bearing. In order that the losses of heat occurring in this way in any two trials should be identical, it was sufficient under the above assumption that this difference of temperature should be the same in both trials, and the temperature of the stuffing-box was regulated to this end by means of the amount of cold water passing into it.

Considerable difference of temperature was observed between the upper and lower brasses of the bearing, and as it seemed probable that the lower one approximated the more closely to the temperature of the shaft, that thermometer was the one used in determining the loss of heat by conduction.

In the later trials I endeavoured to keep the temperatures of the stuffing-box and the bearing at the same level, thus entirely eliminating this cause of loss from the experiments.

Water Jackets for the Low and High Temperature Thermometers.—(Part I., par. 15.)

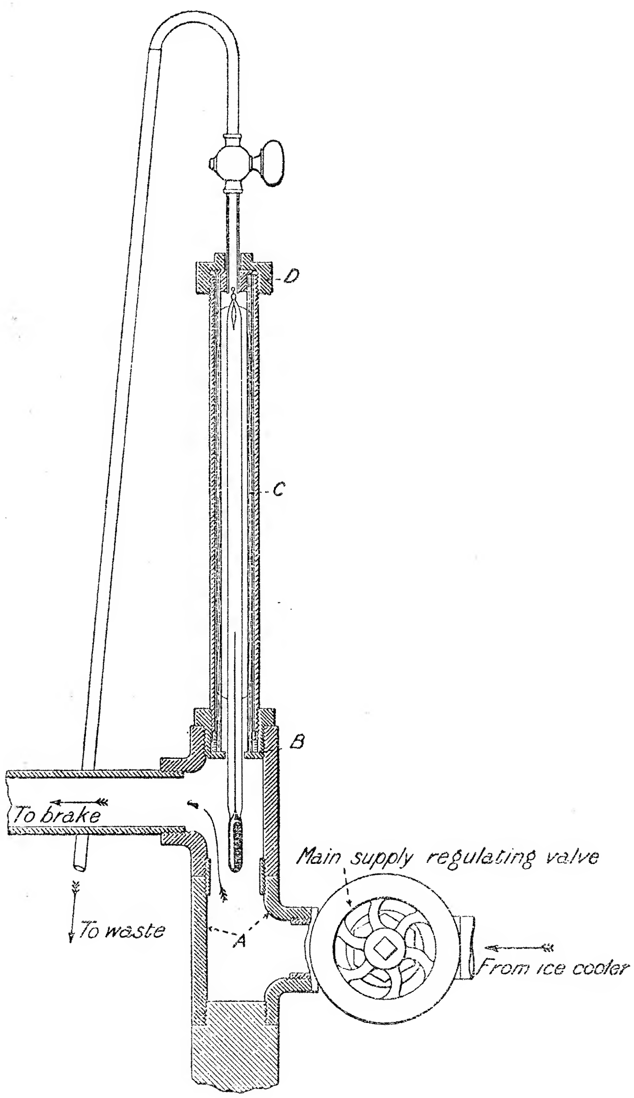
8. It was evident that the temperatures of the water would be much more easily and accurately taken if the whole stem of each thermometer was kept at one temperature. To this end each of the principal thermometers was completely jacketted with a stream of the water whose temperature was required.

The arrangements adopted for this purpose are illustrated in the annexed sketches. (Figs. 3 and 4.)

After leaving the main regulating valve the cold supply water entered a vertical brass T, shown at A. The main volume of the water flowed on to the brake through the horizontal arm of this T. At its upper end the T carried a small stuffing-box, B, into which was fixed a vertical $\frac{1}{2}$-inch diameter glass tube, C. This tube was closed at its upper end by means of a rubber stopper, held in place by the brass cap, D, screwed on to the upper end of a $\frac{3}{4}$-inch slotted copper pipe surrounding

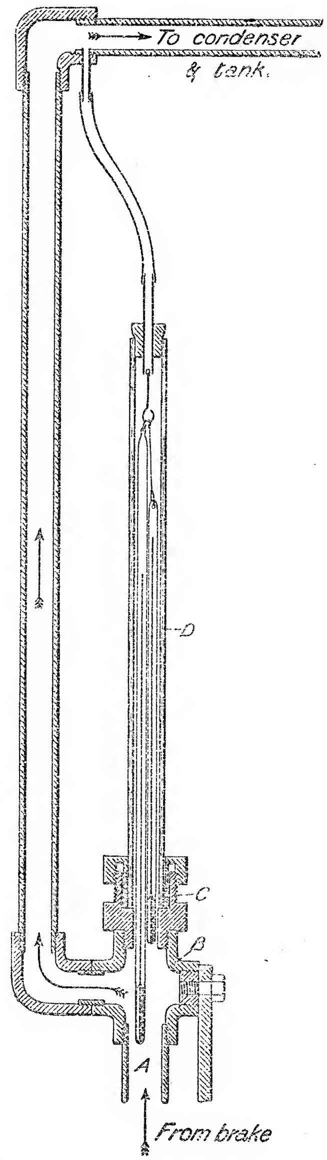
the glass tube. The stopper and cap were both penetrated by a short length of $\frac{1}{8}$-inch diameter brass tube, which carried a gas cock at its upper end. The thermometer was hung by a piece of string from the lower end of the $\frac{1}{8}$-inch pipe—the graduated part of the stem being all clearly visible through the glass walls of the chamber while the bulb was well in the main stream of water flowing through the brass T.

Fig. 3.



Cold water thermometer jacket.

Fig. 4.



Hot water thermometer jacket.

A small stream of water was allowed to run to waste through the small gas cock at the top, thus ensuring the whole of the stem of the thermometer being kept at the proper temperature.

The hot water discharged by the brake flowed from the bent rubber tube, previously mentioned, into the lower end of the vertical 1-inch diameter copper pipe, A. This pipe carried a brass cross, B, at its upper end, while fitted to the top of the cross was the stuffing-box, C, in which was fixed a piece of $\frac{3}{4}$-inch diameter glass tubing, D, forming the thermometer chamber. The upper end of this chamber was closed by a rubber stopper penetrated, as before, by a piece of $\frac{1}{8}$-inch diameter brass pipe, connected by a piece of rubber tubing to the main discharge pipe above.

The left arm of the cross carried an upward-turning elbow, and that again a $\frac{3}{4}$-inch diameter copper pipe, up which most of the water flowed.

The thermometers, two of which were used, were hung to the lower end of the $\frac{1}{8}$-inch pipe in the rubber stopper, so that the bulbs were immersed in the whole stream of water flowing up the 1-inch copper pipe from the brake. One of these thermometers was only used as a finder to indicate the temperature of the water as it rose after first starting the engine, and no record of its readings was kept.

The Condenser.—(Part I., par. 18.)

9. In order that there should not be a large loss of water before weighing, by evaporation from the tank into which it flowed from the brake, it was necessary to cool the stream to a temperature approaching that of the atmosphere.

For this purpose a condenser was constructed after the ordinary chemical pattern. It consisted of a length of 21 feet of $\frac{3}{4}$-inch diameter pipe inserted in an equal length of $1\frac{1}{4}$-inch diameter iron pipe.

Stuffing-boxes were used to form the joints between the two pipes. The hot water from the brake flowed through the inner tube, while a supply of condensing water flowed in the opposite direction through the annular space between the two pipes. By means of this condenser the water entering the tank was always cooled at least to 100° Fahr., and to lower temperatures in the earlier experiments when the water available in the mains was considerably colder.

The Rising Pipe.—(Part I., par. 21.)

10. The thermometer indicating the discharge temperature often gave readings more or less above 212° Fahr.

To provide against any fall in temperature at the thermometer bulb, which might occur by reason of the formation of bubbles of steam in the water, it was found desirable to keep some pressure on the water at that part of its course.

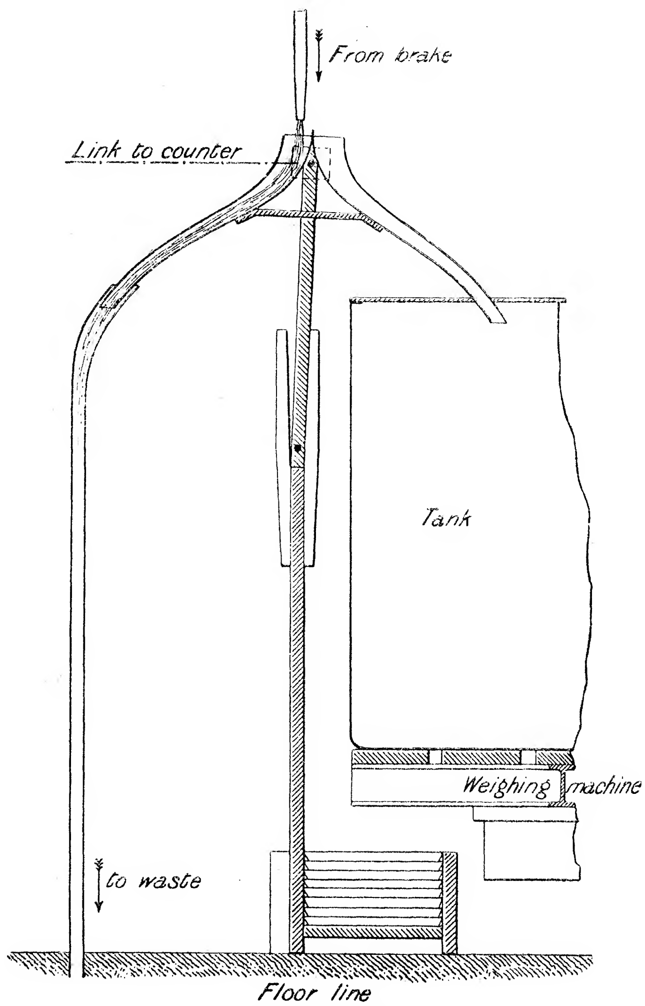
Accordingly, instead of discharging the water directly from the condenser into the tank, it was conducted up a vertical pipe, which was open at the top through a T to the atmosphere. The water then drained down another pipe provided with a nozzle at its lower end, opening into the two-way switch, to be described later. By

this means a head of 11·3 feet of water was maintained at the thermometer bulb, and at a temperature of 220° Fahr. I had not much trouble with bubbles of vapour.

The Two-way Tipping Switch.—(Part I., par. 16.)

11. This was constructed to provide a means of rapidly diverting the water at will, either to waste or into the tank. It consisted, as shown in the sketch, of two curved copper pipes of rectangular section, meeting at their upper ends at an angle of about 30°. Their common side was produced for about $\frac{3}{4}$ inch, and formed into a knife-edge, separating the two orifices.

Fig. 5.



Tipping Switch.

These pipes were rigidly connected to a wooden link which worked about a horizontal axis, distant 25 inches below the knife-edge. Wooden stops were provided to limit the swing of the switch to rather less than 2 inches. One arm of the

switch worked in a funnel forming the top of a pipe leading to waste, while the other worked through a hole in the cover of the tank. The whole arrangement was fixed so that when in the central position the knife-edge was $\frac{1}{4}$ inch vertically below the nozzle at the end of the discharge pipe.

This switch worked exceedingly well, diverting the stream of water almost instantaneously, without making any perceptible splash.

In the later trials this switch was connected by a chain of links with the revolution counter, so that when the latter was pushed into gear with the engine shaft the switch simultaneously directed the water into the tank, and *vice versa*.

Weighing Machine and Tank.—(Part I., par. 13.)

12. To facilitate the weighing, the stream of water was led during each experiment into a galvanized iron tank which stood on the platform of a weighing machine. The tank was 4 feet long by 2 feet 9 inches deep, by 2 feet 9 inches wide. During the experiments it was kept covered by a lid of thin boards, steeped in paraffin wax. These boards were always weighed with the tank, so that any water they might absorb was accounted for. A $2\frac{1}{2}$-inch valve in the tank bottom was used for discharging the water after weighing.

The weighing machine was graduated up to 2200 lbs., and was supplied with three rider weights.

No. 1, the largest, was provided with a knife-edge which fitted into grooves cut in the lever of the machine, each division representing 100 lbs.

No. 2 worked on another scale on the lever, each division representing 1 lb., and graduated up to 100 lbs.

No. 3 was made by Mr. FOSTER, in the laboratory, and indicated 0·01 lb. per division of the second scale. The lever was $32\frac{1}{2}$ inches long, and readings were taken only when the middle of the swing of a pointer fixed to the end of the lever coincided with a line marked on a brass plate alongside it.

It was quite easy in each individual weighing to set the machine to 0·01 lb., but owing, no doubt, to shifting of the platform, levers, &c., I do not think the readings taken were reliable beyond the $\frac{1}{50}$th of a lb.

This machine was not at first quite as sensitive as was necessary to attain the high degree of accuracy required for the purposes of the research. On examination this was found to be due to the slightly imperfect adjustment of the knife-edges attached to the graduated lever. The fault was rectified by Mr. FOSTER, and since then the performance of the machine has been highly satisfactory.

The Rubber Pipe Connections to the Brake.

13. On account of the very considerable pressure to which all the fittings of the

brake were subjected, it was found necessary to bind with tape the rubber pipes supplying the water to ensure them against bursting.

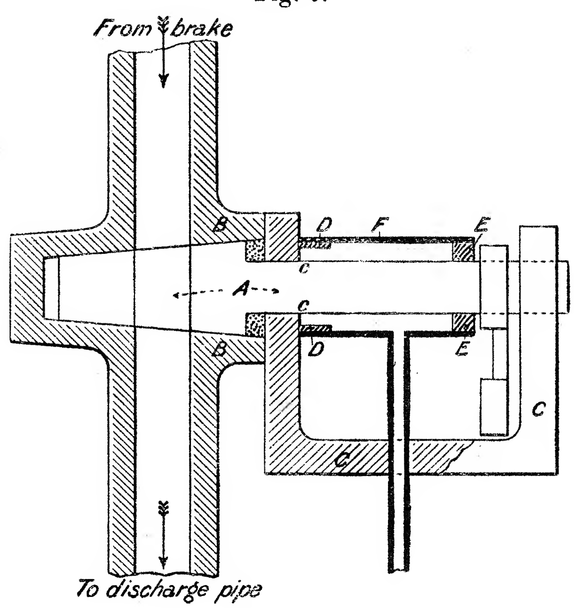
The extra stiffness thus given to these pipes did not much affect the free working of the brake, since none of them had a leverage of more than 4 inches from the centre of the shaft.

The case was, however, different with the bent rubber connection between the brake and the discharge pipe, since in this case the leverage is about 1 foot 6 inches. This pipe was eventually inserted in a cage consisting of a spiral of copper wire, $1\frac{1}{4}$ inches in diameter, through the coils of which were threaded two longitudinal wires to prevent elongation of the cage and rubber tube. By this arrangement the flexibility of the rubber tube was almost unimpaired.

The Device for Catching the Leakage at the Bottom Regulating Cock.—
(Part I., par. 36.)

14. It was found impossible to prevent leakage taking place, generally to a small extent, from the automatic cock controlling the amount of water in the brake. It was, therefore, necessary to provide some means of catching this water, and it was very important that no impediment should be placed in the way of the free working of the cock spindle.

Fig. 6.



A tight joint was made between the valve seating, B, and the bracket, C, which carried the overhanging end of the valve, A. All the leakage, therefore, occurred along the valve spindle at *cc*. The method adopted to catch it was to solder a brass ring on to the bracket at D, and fit a ring of cork of the same diameter tightly on

to the spindle at E. A piece of thin rubber tubing, F, was bound tightly to the ring, D, and the cork, E.

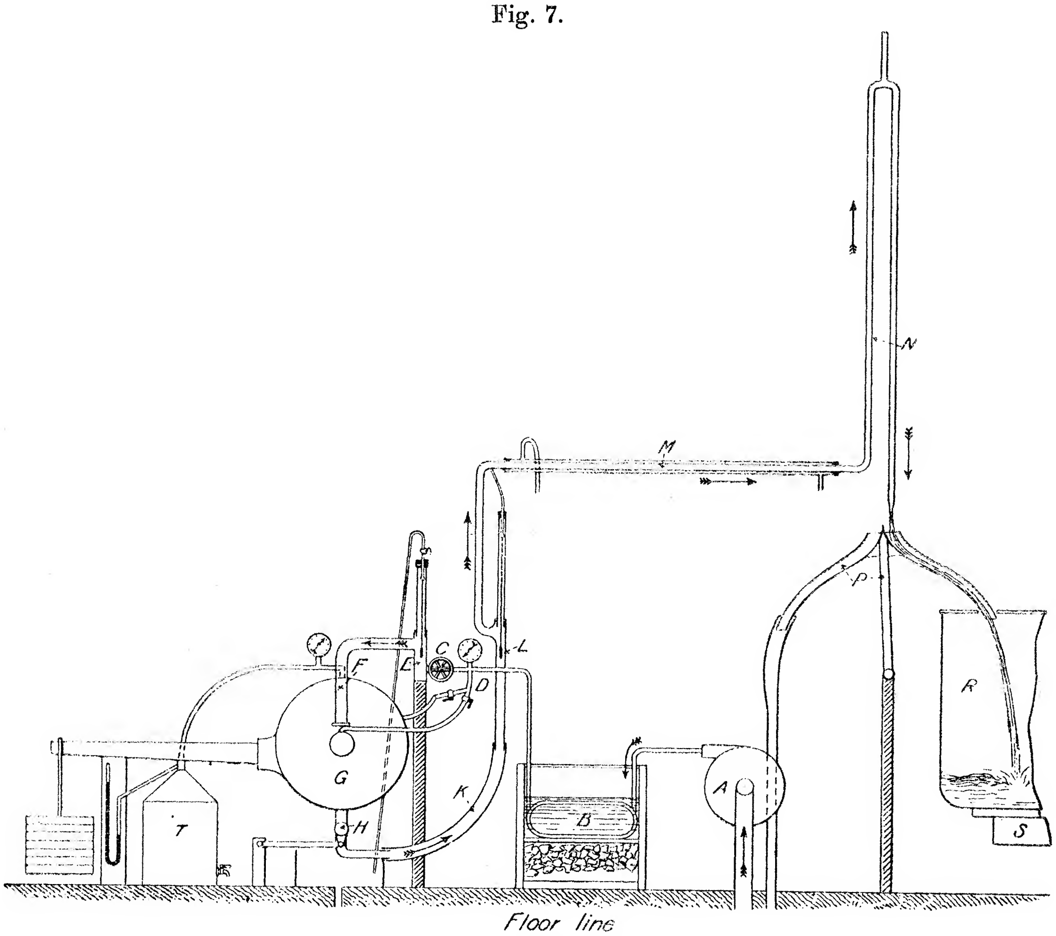
This tube caught all the leakage, which then drained down the smaller tube (shown in the sketch) into a bottle standing on the floor.

To prevent evaporation, the end of this small tube contained a short length of glass tube, the capillarity of which always kept the end closed by a bead of water.

General Arrangement of the Final Apparatus.

15. The general arrangement of the apparatus, as finally set up, is shown in the plates attached to Professor REYNOLDS' paper (Part I.), and in the annexed diagram. The course of the water was as follows :—

Fig. 7.



Final apparatus.

It was drawn from the mains by the circulating pump, A, and forced through the ice cooler, B, to the main regulating valve, C. Between the ice cooler and this valve there was a Bourdon pressure gauge and a branch-pipe, D, supplying water to the

bearings of the brake. Entering the vertical stand pipe, E, the water flowed round the bulb of the initial temperature thermometer, a small stream being diverted to waste through the jacket. The straight flexible rubber pipe, F, then led the stream to the brake, G, from which the water flowed through the automatic valve, H, and the bent rubber pipe, K, to the vertical stand pipe, L, carrying the thermometer for measuring the temperature of discharge. Then passing through the condenser, M, and the rising pipe, N, the two-way switch, P, directed the water either to waste or into the tank, R, standing on the platform of the weighing machine, S. At T is shown the tin vessel forming the artificial atmosphere. A small Bourdon gauge was fitted on to the top of the brake because the mercury gauge, indicating the pressure in the air-vessel, was not visible to the observer when taking readings of the thermometers, and it was important that this pressure should be kept constant.

The Hand Brake and Speed Indicator.—(Part I., par. 30.)

16. In addition to the separate parts of the apparatus already mentioned there was a hand brake by which a moment of about 50 ft.-lbs. could be gradually applied to the engine shaft, and by this means a delicate adjustment of the speed of revolution was obtained.

To make this speed evident a small speed gauge was driven by a gut band from the engine shaft. It consisted of a paddle rotating about a vertical spindle in a cylindrical case. The case contained coloured water, and the pressure generated forced a column of the water up a glass tube, to a height which varied with the speed of revolution.

In Part I., Professor REYNOLDS has referred in one or two instances to the excellent manner in which various parts of the apparatus were constructed by Mr. FOSTER, to whom my thanks are also due for the valuable assistance he often rendered at critical moments in the research, and further for the advice and help he was always willing to give in the construction of apparatus for which I was mainly responsible.

The Method of conducting the Experiments finally adopted—using the Completed Apparatus.

17. During the progress of the experiments, I had at my disposal the services of two men and a boy. Of the men, the first, Mr. J. HALL, was fully engaged in attending generally to the needs of the engine and boiler, and had besides to maintain the boiler pressure at a point which ensured the steady running of the engine. I am bound to state that very much of the success met with must be attributed to the very admirable manner in which Mr. HALL's part of the work was performed.

The duties of the second assistant Mr. J. W. MATTHEWS consisted in regulating the engine speed by means of the hand brake, more particularly at the commencement and end of each trial, and also in keeping a constant pressure of 9 inches of mercury in the artificial atmosphere.

The boy's time was occupied in breaking up the ice and feeding it as required into the ice cooler.

In the last series of experiments three similar trials of 62 minutes duration each were made per day, and the engine having been once started was not stopped till the three trials were completed. Consequently what I say below as to the starting of the engine does not refer to every trial, for after emptying the tank at the close of any one all the necessary adjustments were ready made for the next.

I. The pump and engine were started simultaneously, the brake being therefore supplied with a stream of cold water through the ice cooler. The brake then automatically adjusted the weight of contained water till the load floated clear of the engine floor. The speed was then adjusted till the speed indicator gave the required reading, viz., in all recorded trials 300 revolutions per minute.

II. Since all the work done was expended on the stream of water passing through the brake, its final temperature rose more or less quickly, and by adjusting the regulating valve on the supply pipe the temperature of discharge finally remained steady at 212° Fahr. nearly. In the meantime the supply of water to the stuffing-box was regulated till the temperature of the cover was at the required level.

These adjustments took from a quarter to half an hour, and when made, the engine was allowed to run for some half-hour longer to ensure a steady condition being attained.

The water supply to the condenser had also been regulated till the stream of water issuing from the rising pipe and flowing to waste had the requisite temperature.

III. Readings were then taken of—

(*a*) The revolution counter.

(*b*) The weight of the empty tank and its cover.

IV. When a steady condition was reached, the revolution counter at a given signal was pushed into gear with the small spindle previously mentioned, making connection through the cap with the engine shaft, and simultaneously the two-way tipping switch, which had hitherto been directing all the water to waste, was pulled over and diverted the whole stream into the tank. In the later trials all leakage that did sometimes take place from the stuffing-box, and a slight leakage that always occurred at the automatic cock below the brake, were collected in two bottles kept for that purpose. These were put under the drain pipes in each case as soon as possible after the signal.

The speed of the engine as indicated by the gauge was read when the signal was given, and as soon as possible afterwards a reading was taken of the temperature in the discharge pipe.

V. At intervals of two minutes thirty observations were then taken of the temperatures of supply and discharge of the water to and from the brake, and also at each of these intervals a note was made of the reading of the speed gauge.

At intervals of four minutes fifteen observations were made of a thermometer registering the temperature of the room. Also at intervals of eight minutes readings were taken of the two thermometers in the stuffing-box and on the main bearing.

VI. When sixty-two minutes had elapsed the counter was freed from the shaft, at the same time the water being again diverted to waste.

The drain pipes from the stuffing-box and cock were removed from their respective bottles.

Readings were taken of the speed indicator and of the temperature of discharge.

VII. Fresh observations were made of—

(*a*) The reading of the revolution counter.

(*b*) The weight of the tank and water received during the trial, to which had been added the water caught from the regulating cock.

A record was also made of—

(*c*) The weight of water which had been caught from the stuffing-box.

18. These observations were afterwards reduced as follows :—

Let T_1 = mean temperature of water supplied to the brake.
T_2 = ,, ,, discharged by the brake.
W_1 = weight of tank and contents before the trial.
W_2 = ,, ,, after the trial.
w = weight of water caught from the stuffing-box.
t = rise of reading of the thermometer in the discharge pipe during the trial.
T_S = mean temperature of the stuffing-box cover.
T_B = ,, ,, lower brass of the main bearing.
T_A = ,, ,, air.
N_1 = reading of revolution counter before the trial.
N_2 = ,, ,, after the trial.
M = moment in ft.-lbs. carried by the brake.

Therefore we have for the total heat generated

$$H = (W_2 - W_1)(T_2 - T_1) + w(T_S - T_1) + t \, . \, X \, . + (T_S - T_B) C + (T_2 - T_A) R.$$

The determination of the quantity X and of the constants C and R, representing the losses by conduction and radiation will be dealt with later (pars. 30, 43 and 45).

Also the total work done

$$U = 2\pi (N_2 - N_1)(M + m),$$

where m = error in balance of the brake. This error will be dealt with subsequently (par. 29).

If the capitals H and U refer to trials with a large turning moment on the brake, and the small letters h and u refer to trials with a small turning moment, then for our value of the mean specific heat of water in mechanical units we have

$$K = \frac{U - u}{H - h}.$$

This quantity K is not strictly the same as the mechanical equivalent of heat, of which other determinations have been made, since we are here dealing with the mean specific heat of water between freezing and boiling-points.

For this reason it has been decided not to use the usual symbol J, at any rate at this stage of the research.

19. As an illustration of the method of tabulating and reducing the observations, I append all that were taken in trials 69 and 72 made on the 7th and 8th July, 1896, respectively.

It will be seen that all the observations of temperature, together with the readings of the speed indicator, which were made during the actual progress of each trial, are given on pages 373 and 375 respectively.

With the exception of the two readings of the speed indicator taken at the moments of starting and finishing each trial, and shown in brackets at the top and bottom of column No. 8, I was personally responsible for all observations recorded. These two observations were made by the assistant in charge of the band brake and artificial atmosphere.

In the tables of temperature and speed observations

Col. 1 gives the times at which observations became due, the whole period of 62 minutes being divided into 31 two-minute intervals.
Col. 2 gives the temperatures of supply of the water to the brake.
Col. 3 ,, ,, discharge of the water from the brake.
Col. 4 ,, ,, the air in the engine room.
Col. 5 ,, ,, the stuffing-box cover.
Col. 6 ,, ,, the lower brass of the main bearing.
Col. 7 ,, fall of temperature between the stuffing-box and bearing, being the difference of Cols. 5 and 6.
Col. 8 gives the readings of the speed indicator.

Observations of the revolution counter and of the weight of the tank before and after each trial, are given on pages 372 and 374 respectively.

As I had to take all the observations myself, it was, of course, impossible to make them simultaneously at the times indicated in Col. 1. They were, however, always taken in the same order, as follows.

When the time for the next ensuing series of observations had arrived as given by a watch lying on the table at my side, I immediately read the temperatures of

supply and discharge and the speed gauge in the order named, and after reading the three I entered them in the note-book. This generally took about a quarter of a minute. If then a reading of the atmospheric temperature was due, it was next taken and entered. After that the temperatures of the stuffing-box cap and of the bearing were noted in their turn, the whole series of observations being made in 1 or $1\frac{1}{4}$ minutes.

The interval which then elapsed before the next series of observations became due was often fully occupied in making adjustments of the regulating valve controlling the main water supply to the brake; of the cock regulating the supply to the stuffing-box; and of the speed of the turbine driving the pump, small alterations at all these points being frequently necessary.

At the head and foot of Cols. 3 and 8 will be seen observations in brackets. These observations were taken at the moments of starting and ending the trials, and were required in the calculation of a terminal correction to be referred to later.

At the close of each trial a mean of the observations occurring in Cols. 2, 3, 4, 5 and 7 was made, the two observations in brackets in Col. 3 being omitted in calculating these means.

On pages 372 and 374 additive corrections to the weights and to the mean temperatures of supply and discharge are given. These will be referred to later.

It will be noticed that in neither of the trials chosen was there any leakage of water from the stuffing-box.

The observations are given again in the partially reduced form which has been adopted for the final tabulation of the results on p. 376.

Cols. 1 to 8 should be self-explanatory.

Col. 9 gives the first approximation to the heat generated, obtained by multiplying the weight of water by its mean rise in temperature.

Col. 11 gives the difference of the temperature of the stuffing-box (supposed to be a measure of that of the water leaking from it), and the temperature of supply.

Col. 12 gives the loss of heat due to this leakage, and represents the product of Cols. 10 and 11.

Col. 13 gives the rise of temperature of the brake during the trial, and is assumed to be equal to the difference of the two temperatures given in brackets in the table of temperature observations (Col. 3).

Col. 14 gives the terminal correction to the heat required on account of the increase of heat in the brake itself during the trial.

Col. 15 gives the difference between the mean temperature of the stuffing-box and of the shaft bearing. As already explained the loss of heat by conduction has been assumed proportional to this difference, and a determination of its amount will be given later. At present it is

sufficient to say that a loss of 12 thermal units occurred per trial per unit fall of temperature along the shaft.

Col. 16 gives, therefore, the product of this difference × 12, which represents the total loss by conduction.

Col. 17. The difference of temperature between the brake and the surrounding air was taken as being equal to the difference of the mean discharge temperature of the water and that of the air. The determination of the constant representing the loss of heat per unit difference of temperature is given later, and consequently,

Col. 18 gives the product of this constant × the difference of temperature in Col. 17.

Col. 19 gives the sum of the heat in Col. 9 added to all the corrections afterwards given.

A further Table (p. 376) gives the work done, and the corrected values of the heat generated in these two trials, and the differences between them.

The value of K in the last column is then found by dividing the difference of work in Col. 4 by the difference of heat in Col. 6.

A slight inaccuracy has been pointed out to me by Professor REYNOLDS in the method of finding the mean temperatures of supply to and discharge from the brake. It was originally intended that the trials should be of exactly one hour's duration, and that the first series of readings should be taken one minute after the start. It was found impossible to do this, on account of the number of points requiring attention in the first few minutes, and consequently I made all trials 62 minutes long, and took the first reading two minutes after starting. The mean used has not therefore been obtained strictly in accordance with the middle breadth rule. Any error introduced would be of the occasional type, and should be eliminated in the mean of a number of trials.

July 7, 1896.

Trial No. 69 (A).

Moment on the brake . . . . 600 ft.-lbs.

Trial began at 11.17 A.M., and ended at 12.19 P.M

Reading of revolution counter after trial	92,948.
„ „ „ before trial	75,400.
Number of revolutions during trial	17,548.

Weight of tank and water after trial	811·94 − ·5 lb.
„ „ „ before trial	342·16 + ·4 „
Weight of water discharged by brake during trial, including leakage from bottom cock	468·88 lbs.

Mean temperature of water in the discharge pipe .	212·007° F. + ·04.
„ „ „ supply pipe . .	33·595° − ·52.
Mean rise of temperature of the water	178·972° F.

Weight of water caught from stuffing-box . . . . 0 lb.

Temperature of water entering the tank . . . . = 100° F.

1.	2.	3.	4.	5.	6.	7.	8.
Times.	Temperatures. Water supplied to brake.	Water discharged from brake.	Air.	Stuffing-box cover.	Lower brass of bearing.	Fall of temperature between stuffing-box and bearing.	Readings of speed-gauge (revolutions per minute).
	°	°	°	°	°	°	
Began 11.17	..	(212)	..	..	..	..	(302)
19	33·57	211·9	74·4	..	..	..	300
21	33·5	212·0	..	..	..	..	302
23	33·57	212·3	75·7	107	107	..	302
25	33·58	211·3	..	..	..	..	303
27	33·58	211·5	76·0	..	..	..	302
29	33·58	212·2	..	..	..	..	304
31	33·57	211·1	76·4	109	110	−1	302
33	33·6	211·0	..	..	..	..	299
35	33·6	211·0	76·5	..	..	..	299
37	33·6	214·9	..	..	..	..	303
39	33·6	213·7	77·5	109	111	−2	301
41	33·62	213.3	..	..	..	..	301
43	33·6	213·2	76·8	..	..	..	299
45	33·59	212·2	..	..	..	..	301
47	33·64	211·5	77·0	110	111	−1	301
49	33·62	211·8	..	..	..	..	303
51	33·64	212·0	78·1	..	..	..	304
53	33·59	212·3	..	..	..	..	299
55	33·59	212·1	76·5	110	111	−1	301
57	33·58	212·2	..	..	..	..	301
59	33·6	211·8	77·8	..	..	..	301
12.01	33·62	211·9	..	..	..	..	302
3	33·61	212·0	78·3	115	113	2	301
5	33·62	211·5	..	..	..	..	300
7	33·6	212·0	79·0	..	..	..	300
9	33·57	211·6	..	..	..	..	300
11	33·59	211·6	76·8	112	113	−1	297
13	33·57	211·5	..	..	..	..	300
15	33·6	211·3	77·1	..	..	..	301
17	33·66	211·5	..	..	..	..	301
Ended 19	..	(212)	..	..	..	..	(302)
Means . . .	33·595	212·007	76·9	110·3	..	−·57	..

July 8, 1896.

Trial No. 72 (A).

Moment on the brake	1200 ft.-lbs.

Trial began 11.11 A.M., and ended 12.13 P.M.

Reading of revolution counter after trial	146,311
,, ,, ,, before trial	129,000
Number of revolutions during trial	17,311

Weight of tank and water after trial	1283·50 — 1·31 lbs.
,, ,, ,, before trial	347·21 + ·4 lb.
Weight of water discharged by brake during trial, including leakage from bottom cock	934·58 lbs.

Mean temperature of water in the discharge pipe .	212·46° F. + ·04
,, ,, ,, supply pipe .	34·706° — ·55
Mean rise of temperature of the water	178·344° F.

Weight of water caught from stuffing-box . . . . = 0 lb.

Temperature of water entering tank . . . . = 101° F.

1.	2.	3.	4.	5.	6.	7.	8.
	Temperatures.					Fall of temperature between stuffing-box and bearing.	Readings of speed-gauge (revolutions per minute).
Times.	Water supplied to brake.	Water discharged from brake.	Air.	Stuffing-box cover.	Lower brass of bearing.		
Began 11.11	..	(212·4)	..	..	..	..	(300)
13	34·74	212·3	72·0	..	..	..	302
15	34·8	211·5	..	..	..	..	300
17	34·71	212 8	73·7	97	99	— 2	304
19	34·7	212·9	..	..	..	..	303
21	34·69	211·7	74·0	..	..	..	299
23	34·72	212·0	..	..	..	..	302
25	34·7	212·6	73·3	101	101	..	303
27	34·77	212·8	..	..	..	..	307
29	34·78	213·5	74·4	..	..	..	302
31	34·77	214·0	..	..	..	..	300
33	34·69	213·2	74·7	101	102	— 1	301
35	35·0	213·2	..	..	..	..	299
37	34·6	214·0	75·6	..	..	..	303
39	34·7	214·4	..	..	..	..	307
41	34·76	214·0	74·7	104	103	1	302
43	34·79	212·8	..	..	..	..	304
45	34·66	213·0	74·8	..	..	..	301
47	34·75	212·3	..	..	..	..	300
49	34·66	211·6	75·7	105	104	1	297
51	34·68	211·2	..	..	..	..	302
53	34·68	212·0	75·4	..	..	..	302
55	34·66	211·6	..	..	..	..	299
57	34·66	211·0	75·3	104	105	— 1	297
59	34·58	211·3	..	..	..	..	302
12.1	34·6	212·3	76·0	..	..	..	305
3	34·59	212·9	..	..	..	..	299
5	34·67	211·8	76·0	107	106	1	301
7	34·7	211·4	..	..	..	..	302
9	34·69	211·9	75·8	..	..	..	304
11	34 68	211·8	..	..	..	..	302
Ended 13	..	(211·6)	..	..	..	..	(300)
Means . . .	34·706	212·46	74·8	102·7	..	— 0·14	

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.
Date.	Trial No.	Time of start.	Moment (ft.-lbs.)	No. of revolutions of engine shaft.	Work done (ft.-lbs.)	Weight of water discharged by brake (lbs.)	Rise of temperature in the brake (° F.)	Heat generated, less losses due to radiation, &c. (B.T.U.)	Weight of water caught from stuffing-box (lbs.)	Rise of temperature in the brake (° F.)	Loss of heat by leakage (B.T.U.)	Rise of temperature of brake during trial (° F.)	Terminal correction to heat (B.T.U.)	Fall of temperature along shaft between stuffing-box and bearing (° F.)	Loss of heat by conduction (B.T.U.)	Difference of temperature between brake and air (° F.)	Loss of heat by radiation (B.T.U.)	Corrected heat (B.T.U.)
7 July, '96	69	11. 7 A.M.	600	17,548	66,154,556	468·88	178·972	83,916	..	..	..	..	..	—0·57	—7	135·1	1078	84,987
8 July, '96	72	11.11 A.M.	1200	17,311	130,522,170	934·58	178·344	166,677	..	..	..	—0·8	—46	—0·14	—2	137·7	1099	167,728

1.	2.	3.	4.	5.	6.	7.
Determination No.	Trial No.	Works.	Diff. of works.	Heats.	Diff. of heats.	K.
	72 69	130,522,170 66,154,556	.. 64,367,614	167,728 84,987	.. 82,741	.. 777·95

The Barometer.—(Part I., par. 47.)

20. Before dealing with the thermometers and their corrections, it becomes necessary to describe a combined barometer and manometer which was constructed to measure the pressures of steam employed in the determination of the boiling-points on the thermometer used to measure the discharge temperature.

The structural details of this instrument are given in Professor REYNOLDS' paper. At present it is sufficient to say that it consisted of a cast-iron, bottle-shaped reservoir, through the neck of which the glass tube holding the mercury column was carried in a stuffing-box, which made a perfectly air-tight joint between the glass and the reservoir. The pressure to be measured was introduced through a small iron pipe, which penetrated horizontally the cast-iron wall of the reservoir, and then turned vertically upwards till its open mouth stood above the level of the mercury inside. Two circular plate-glass windows in the reservoir walls provided a means of ascertaining the level of the mercury surface. In order to measure the height of the mercury column supported by any external pressure, a brass sleeve was made, which fitted outside the glass tube and the upper part of the reservoir. This sleeve consisted of a piece of $\frac{3}{4}$-inch diameter brass pipe fixed into a conical brass casting, which carried a truly-turned bevelled edge at its lower extremity. This conical casting engaged by an internal screw of twenty threads to 1 inch with the neck of the cast-iron reservoir. The upper part of the sleeve carried an internal thread of the same pitch, and into this was screwed a second piece of pipe through which two long narrow slits were cut at opposite extremities of a diameter. A third piece of brass pipe engaged with the upper end of the piece just mentioned, and was provided at its lower end with a truly-turned bevelled edge.

In use the bevelled edge on the conical brass casting was first adjusted to the surface of the mercury in the reservoir, and then the upper bevelled edge was adjusted to the surface at the top of the mercury column. Suitable horizontal and vertical scales were provided to enable me to measure the vertical distance between these two bevelled edges to $\frac{1}{1000}$ of an inch.

It was necessary to standardise this scale (Part I., par. 44). There is a Whitworth measuring machine in the laboratory, which is provided amongst others with standard end gauges of 9 inches and 3 inches long respectively.

Two new steel standards were made by Mr. FOSTER as nearly as possible of the same length as the 9-inch Whitworth, and by means of the measuring machine I determined their exact lengths as follows, three comparisons being made of the two new gauges with the standard. The table shows the readings obtained.

	Whitworth standard 9-inch gauge.	Laboratory standard gauge, No. 1.	Laboratory standard gauge, No. 2.
Readings on divided wheel of machine	0·0011 0·00112 0·00114	0·00105 0·0010 0·00097	0·00095 0·0009 0·00098
Mean readings . .	0·00112	0·001007	0·000943
True lengths . . .	9 inches	9 inches − 0·000113	9 inches − 0·000177

These three 9-inch standards, together with the 3-inch Whitworth, therefore gave a length when placed end to end of

$$30 \text{ inches} - 0{\cdot}00029 \text{ inch}.$$

The next operation was to construct a single steel standard with a length of approximately 30 inches. This bar being made, and the measuring machine not being long enough to accommodate 30 inches, the measurements were made between the centres of a large lathe in the laboratory. Two centres were made with polished flat ends. The one was put in the fixed headstock, while the second was carried by the movable sleeve of the loose headstock which had previously been securely bolted to the lathe bed in a convenient position. A temporary wooden trough was made to carry our four short standards, and correctly line them between the two centres. The reciprocating centre in the loose headstock was then gradually screwed up till the gravity piece of the measuring machine just floated between the end of the adjacent standard and the centre. A mark on the hand-wheel actuating the centre was then fixed by means of a pointer. The four standards were then removed, and the 30-inch bar substituted for them, and the operation of bringing up the centre repeated. The circumferential distance then separating the pointer from the mark on the hand-wheel was then carefully measured.

A series of five of these observations were made, and the following readings taken, viz:—

(1) $-$ 0·1 inch (3) $+$ 0·09 inch (5) $+$ 0·03 inch
(2) $-$ 0·05 inch (4) $+$ 0·02 inch.

$$\text{Mean} = -0{\cdot}002 \text{ inch}.$$

The hand-wheel had a diameter of $9\frac{1}{4}$ inches, and was fixed to a screw of $\frac{1}{5}$-inch pitch.

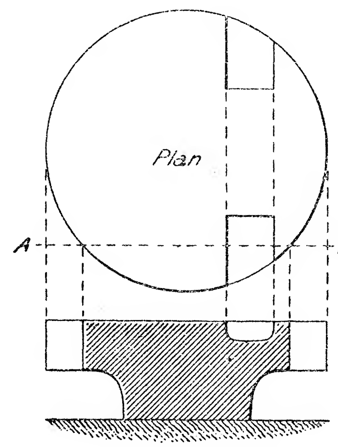
The 30-inch bar was therefore short of the length of the four steel standards by 0·0000138 inch.

Its correct length was, therefore,

$$30 \text{ inches} - 0{\cdot}0003 \text{ inch.}$$

As the barometer was only graduated to 0·001 inch, no error was introduced in assuming the bar to be exactly 30 inches long.

(Part. I., par. 48).—For the purpose of transferring this standard 30 inches to the brass sleeve forming the scale of the barometer, a circular cast-iron surface plate was made. This plate had two pieces cut out of it, as shown in the sketch. The plate was fixed with its surface level, and then the brass sleeve was placed centrally upon it, standing upright on its lower bevelled edge. In this position the portion of the surface between the two grooves cut in the plate corresponded exactly to the surface of the mercury in the barometer between the two windows previously mentioned. As it was probable that in actual use the lower bevelled edge would be slightly above the mercury surface, the sleeve was packed up by means of some very fine sheets of tissue paper till a line of light could be seen under it. Four sheets were necessary to effect this; one of these was removed, and then the standard 30-inch bar was placed inside the brass tube, standing with one end on the surface plate. The upper bevelled edge was then adjusted till the line of light between it and the top of the steel standard was obscured, and the scale was made to read 30 inches in that position.



Section AB.

Together with Mr. Foster I made this adjustment a number of times, but after once fixing the 30-inch mark, the reading of the length of the steel standard never varied by as much as 0·0003 inch from 30 inches.

Unfortunately, the comparison was made at a temperature of 67° Fahr., while the standard temperature of the Whitworth gauges was 60° Fahr. A formula of reduction of the readings of the barometer therefore became necessary at all temperatures.

Taking for the coefficient of linear expansion of brass per ° Fahr.	0·000012
,, ,, ,, ,, steel ,,	0·0000066
,, ,, ,, ,, the mercury column of the barometer	0·0001.

Then at 67° Fahr. the true length of the brass barometer scale

$$= 30 \, \frac{1 + 35 \times 0{\cdot}0000066}{1 + 28 \times 0{\cdot}0000066}$$

$$= 30{\cdot}000138 \text{ inches.}$$

To find T, the temperature at which the scale gives correct readings, we have, if $T = t + 32°$,

$$\frac{1 + t \times 0{\cdot}000012}{1 + 35 \times 0{\cdot}000012} = \frac{30}{30{\cdot}000138},$$

which gives $t = 31°$ and $T = 63°$ Fahr.

The coefficient of expansion of the mercury column relative to the brass scale is 0·000088.

Now if H_T = reading of barometer in inches at T° Fahr., and as before

$$t = T - 32,$$

then the corresponding corrected height of the column at a temperature of 63° Fahr.

$$= H_{63} = \frac{1 + 31 \times 0{\cdot}000088}{1 + t \times 0{\cdot}000088} H_T$$

$$= \frac{1{\cdot}002728}{1 + t \times 0{\cdot}000088} H_T,$$

and if H_0 = the corresponding pressure reduced to inches at the freezing-point, then

$$H_{63} = H_0 (1 + 0{\cdot}0031).$$

Therefore for any required pressure H_0 inches at a temperature of 32° Fahr., the corresponding reading at T° Fahr. is

$$H_T = \frac{1 + 0{\cdot}000088t}{1{\cdot}002728} H_0 \times 1{\cdot}0031,$$

or, allowing for the capillarity depression in a half-inch tube, this becomes

$$H_T = (1{\cdot}00037 + 0{\cdot}000088t) H_0 - 0{\cdot}009.$$

This formula has been used throughout to determine the steam pressures required for the verification of boiling-points to be discussed later (pars. 23 and 24).

The Thermometers.

21. The thermometers used for the measurement of the temperatures of supply and discharge of the stream of water passing through the brake were supplied by Mr. J. CASARTELLI of Manchester.

Their indications were read through the glass walls of their respective chambers by eye simply, parallax being avoided by the use of a small mirror placed behind the thermometer in each case.

Freezing-point Thermometers.

22. Two similar thermometers were obtained, one only of which was ever used during the experiments. This was a chemical thermometer, bearing the laboratory mark 2Q, with a $\frac{1}{4}$-inch diameter stem having its scale very plainly etched in black lines on the glass. The length was $11\frac{1}{2}$ inches over all, the bulb being $1\frac{1}{2}$ inches long, and then at a distance of $2\frac{1}{2}$ inches from the top of the bulb the graduations began. The scale extended from 30° to 45° Fahr., $6\frac{3}{8}$ inches of the stem being occupied by the 15° mentioned. Each degree was divided into tenths, and it was easy to estimate to the hundredth of a degree.

The index error of this thermometer was repeatedly checked during the whole period occupied by the research by being immersed in a mixture of pounded ice and water.

The table appended gives the corrections and the dates on which tests were made :—

Date.	Reading.	Correction.
5th December, 1895	31·7	+ 0·3
20th December, 1895	31·71	+ 0·29
9th January, 1896	31·67	+ 0·33
17th January, 1896	31·67	+ 0·33
31st January, 1896	31·57	+ 0·43
5th February, 1896	32·48	− 0·48
20th February, 1896	32·46	− 0·46
16th March, 1896	32·46	− 0·46
21st April, 1896	32·47	− 0·47
25th June, 1896.	32·47	− 0·47
7th July, 1896	32·52	− 0·52

Before making the test on January 31st the hot water from the brake backed up round this thermometer, so that the sudden alteration in the reading is accounted for to some extent.

Also up to this time part of the mercury had remained stuck in the upper bulb, but Dr. HARKER, of the Physical Department, now succeeded in bringing the separated mercury down into contact with the column below.

By permission of Dr. SCHUSTER the scale of this thermometer was compared by Dr. HARKER on the 27th April, 1896, with a standardised thermometer (BAUDIN, No. 12,771) in his possession between the points 32° and 35° Fahr.

This comparison showed that the correction of − 0·47 as obtained on April 21st was correct between 33° and 34°, which was the part of the scale used in most of the experiments up to that date.

At 35°, however, the correction increased to − 0·5, and consequently in the later experiments, when the temperature of supply in the heavy trials approached this

point, a suitable correction was made to that already obtained by immersion in the mixture of pounded ice and water.

Boiling-point Thermometers.

23. In the first instance two similar thermometers were made to order to be ready for use in the discharge tube, but on one of these being broken, two additional ones were obtained. Only one of the four was, however, used in the research, viz., P1.

This was a chemical thermometer with a $\frac{1}{4}$-inch stem, having the scale engraved as already described. The length was $16\frac{1}{2}$ inches over all, the bulb being $1\frac{1}{2}$ inches long, and a blank space of $5\frac{1}{4}$ inches separating the top of the bulb from the first graduation. The scale extended from 200° to 220° Fahr., the 20° occupying $8\frac{3}{8}$ inches of the stem.

During the course of an experiment the reading of this thermometer was continually altering slightly. This fluctuation made it almost impossible to read the temperatures to $\frac{1}{100}$th of a degree. So that only the nearest $\frac{1}{10}$th of a degree has been recorded throughout.

The English standard boiling point, viz., 212° Fahr., is defined to be the temperature of saturated steam under a pressure which would sustain a column of mercury 29·905 inches long at the temperature of melting ice at the sea level in the latitude of Greenwich.

This corresponds exactly, on being corrected for the variation in the value of gravity, to the modern definition of the boiling point on the Centigrade scale, the pressure in this case being equivalent to a column of mercury 760 millims. long in latitude 45°, the other conditions being as before.

It was consequently possible to use REGNAULT's steam table in the neighbourhood of the atmospheric boiling point as a standard of comparison for the scale of this thermometer.

In order to conduct the comparison in Manchester, a knowledge of the relative value of gravity was necessary.

This was deduced from a formula given in 'Mémoires sur le Pendule' (Société Française de Physique), which is given below,

$$\frac{g\phi}{g_{45}} = (1 - 0·00259 \cos 2\phi),$$

where $\frac{g\phi}{g_{45}}$ is the ratio of the value of gravity in latitude ϕ to its value in latitude 45°.

The latitude of Manchester being 53° 29′, this gives

$$\frac{g\phi}{g_{45}} = 1·000756.$$

The altitude of the Owens College, Manchester, has no appreciable effect on the value given by the above formula.

I give below the table of steam pressures used in the calibration of the scale of the thermometer P1.

Temperature on Centrigrade scale.	Temperature on Fahrenheit scale.	Pressure of steam in millims. of mercury reduced to 0° C. and sea level in lat. 45°.	Pressure of steam in inches of mercury reduced to 0° C. and sea level in latitude of Manchester.
99°	210°·2	733·305	28·849
100	212·0	760·000	29 899
101	213·8	787·590	30·984
102	215·6	816 010	32·102

24. The general arrangement of the apparatus used to check the scale of the thermometer P1 will be gathered from the annexed sketch (fig. 8), and from Plate 6 attached to Professor REYNOLDS' paper, (Part I., par. 48).

A is an ordinary copper boiling-point apparatus, the steam from the boiling water passing up an inner tube in which the thermometer to be tested is hung, and then flowing down again so as to jacket this tube, finally escaping into the atmosphere through the cock shown. The top of the inner tube is closed by a cork having two holes, in one of which is fitted a half-inch brass tube for connection with the manometer, the other carrying the thermometer.

B is a glass flask containing an artificial atmosphere, of which the pressure is under control.

C is the combined barometer and manometer used to measure the pressure in A and B.

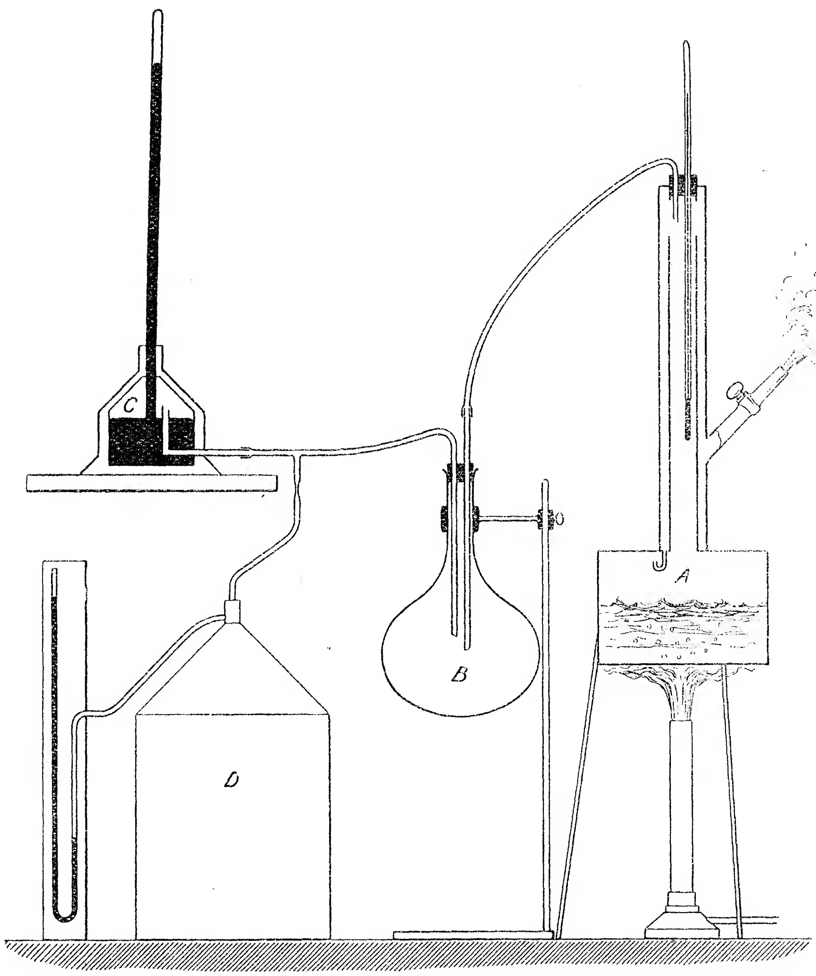
D is the tin receiver previously described, the pressure in which is kept at about 18 inches of mercury, as measured on a U-gauge. This receiver is in free communication through a capillary glass tube with the tube connecting the flask B and the manometer C.

The bore of the capillary tube just mentioned is just sufficient to admit a very small stream of air from the receiver through the flask B, and so out into the atmosphere by way of the cock on the boiler. The object of this stream of air was to counteract the tendency of the steam in the boiler to diffuse down the connecting rubber tube into the flask, where condensation would occur, and possibly some water might get into the barometer, it having been found quite impossible to keep a steady pressure in the apparatus whenever the steam made its way as far as the glass flask, B.

The boiler was well lagged and protected as far as practicable from draughts. A

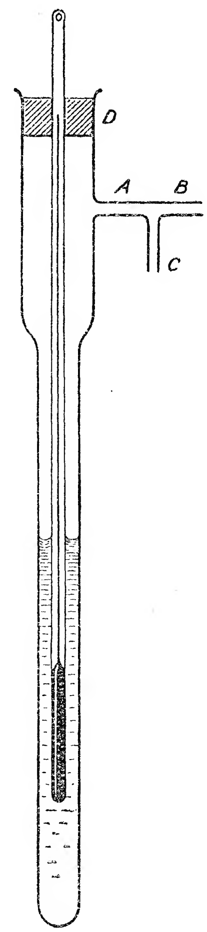
thermometer was hung alongside the brass scale tube of the barometer, and its reading was assumed to be the temperature of the barometer. Allowance having been made for this temperature, the steam escape cock was adjusted till the pressure inside the apparatus, as measured in the barometer, was at the required level. A reading was then taken of the thermometer under examination. The stem was

Fig. 8.



Apparatus for checking boiling-points.

Fig. 9.



pushed as far as possible into the boiler, the reading standing about a quarter inch above the top of the cork. Since there was always some escape of steam which blew up the hole in which the thermometer was inserted, it was not thought necessary to attempt to make any correction for the exposed part of the stem.

The annexed table gives the readings taken from this thermometer when immersed in steam of various known temperatures and the dates on which the tests were made :—

Date.	Readings obtained from thermometer P1 when immersed in steam at temperature			Correction used in experiments.
	212°	213°·8	215°·6	
28 Nov., 1895	211·43	213·26	215·01	+ 0·57
4 Dec., 1895	211·44	213·28	215·03	+ 0·56
5 Dec., 1895	211·5	213·33	215·07	+ 0·5
6 Dec., 1895	211·51 (rising)	..	..	+ 0·48
	211·53 (falling)	..	..	
12 Dec., 1895	At tempera	ture 210°·46 reading	was 210°·05	
9 Jan., 1896	..	213·38 (rising)	..	+ 0·44
		213·40 (falling)	..	
17 Jan., 1896	..	213·49	..	+ 0·34
23 Jan., 1896	..	213·49	..	+ 0·34
31 Jan., 1896	..	213·49	..	+ 0·34
8 Feb., 1896	211·76	213·57	215·3	+ 0·24
20 Feb., 1896	211·78	213·6	215·34	+ 0·22
	At	211°·34 reading was	211°·1	
16 Mar., 1896	211·86	213·66	215·4	+ 0·14
	At	211°·07 reading was	210°·87	
18 April, 1896	..	213·7	215·45	+ 0·11
15 June, 1896	211·94	213·74	215·5	+ 0·06
6 July, 1896	211·96	213·75	215·52	+ 0·04

25. In the case of each of these thermometers, viz., Q2 and P1, the water surrounding them was under a very considerable pressure, and it was therefore necessary to determine the effect of pressure on the reading given by each.

A piece of strong glass tube, fig. 9, about 1 foot in length and $\frac{3}{8}$ inch inside diameter, having one end fused up, was provided with a slightly wider mouth, in which was inserted a small branch pipe, A. This branch again split up into two arms, one of which, B, was connected through a rubber tube with an air receiver in which the pressure was indicated by a U-gauge, while the other, C, communicated directly with the atmosphere. Each of the branches B and C could be closed at will by means of a screw clip on the rubber tubing.

The pressure tube having been about half filled with water, the thermometer under consideration was fixed inside it by means of a cork, D.

In the case of the freezing-point thermometer, Q2, the pressure tube was then surrounded with pounded ice. After the contained water had cooled sufficiently for the thermometer inside to remain steady, the communication with the atmosphere was closed, and the full pressure of the air receiver put on the thermometer bulb by opening the clip on the tube, B. The rise in the reading due to the known rise of pressure was then noted. A number of these observations were made, using different additional pressure in each case. The result obtained was that for a rise in pressure on the bulb due to 1 inch of mercury, the rise in the reading was 0·0072°.

In the case of the boiling-point thermometer, P1, the pressure tube was immersed

in the steam generated in the copper boiler previously alluded to. Similar procedure gave in this case a mean rise of 0·0066° per inch rise of pressure.

After applying corrections (to be dealt with later—par. 62), rectifying the thermometric indications on this account, I think that no error of greater magnitude than 0·01° can have existed in the calculated mean rise of temperature in any trial.

On 180° this gives accuracy of 1 part in 18,000.

26. In addition to the thermometers just dealt with, three others were used, on the readings of which depended the additive corrections to the heat already referred to. One of these indicated the atmospheric temperature, while two others were placed one on the stuffing-box and the other on the shaft bearing.

On the differences of heat which were used as the divisors in the determination of the equivalent from each pair of trials, these corrections all became extremely small quantities, and therefore it was of no importance that small errors should exist in these thermometers. Their scales were therefore never calibrated. Still another thermometer was used to determine the temperature of the stream of water entering the tank. As it was only necessary to keep this temperature in each pair of trials at the same level, errors in this thermometer were negligible.

Weighing Machine and 25-lb. Weights used on the Brake.—(Part I., par. 40.)

27. The absolute value of the unit used in the graduation of the lever of the weighing machine was a matter of indifference, but it was of vital importance that the same unit should be used for the weighing machine and for the 25-lb. weights used on the brake.

A set of iron weights were, however, sent down to the Manchester Town Hall, and there compared with the Board of Trade standards.

The comparison of the 25-lb. weights with our standard 25 lbs. was one of the first things undertaken in the course of the investigation. This was done by first balancing the standard placed on the platform of a small weighing machine in the laboratory by adjustment of the rider weights on the lever of the machine. The standard was then removed, and one of the 25-lb. weights substituted, a balance being made by adding to or drilling out some of the lead inserted in the weight.

This adjustment was accepted as perfectly satisfactory till towards the close of the experiments, when a small difference in the value of the equivalent as derived from trials in which different numbers of the weights were used, seemed to suggest an error in the weights themselves.

Accordingly, on the 9th June, 1896, I again compared the weights with the standard on a temporary balance, consisting of a simple lever with three knife-edges in a straight line, with the following result :—

Weight number.	True weight.
1	25·00
2	25·02
3	25·03
4	25·02
5	25·01
6	24·99
7	25·02
8	25·02
9	25·03
10	25·00
11	25·04
Hanger	24·99

And a lead balance weight to be referred to later, which weighed 13·98 lbs. instead of 13·97 lbs. as assumed

On the 17th of January, 1896, a set of four of these 25-lb. weights, at that time all supposed accurate, were used as a standard 100 lbs., by which a series of corrections to the 100-lb. scale of the weighing machine were obtained. These corrections have been used throughout the investigation, and are given below :—

Reading .	300	400	500	600	700	800	900	1000	1100	1200	1300
Correction	0·4	..	..	−0·12	−0·42	−0·5	−0·65	−1·12	−1·22	−1·31	−1·78

Rider weights Numbers 2 and 3 were at the same time made correct on their whole range.

In June another comparison was made, and the set of four weights, Numbers 2, 8, 9, and 10 were found to give substantially the same list of corrections as previously obtained.

The complete set of weights were then again weighed on the weighing machine, using the list of corrections given, together with the true value of the standard 100 lbs. The result was a verification of the list of their values already given.

The maximum error that might possibly be produced by using the weights on the brake in specially arranged groups was found to be—

In a pair of trials carrying moments of 1200 and 600 ft.-lbs. respectively, − 0·037 per cent. or + 0·043 per cent., and in a pair of trials run with moments of 1200 and 400 ft.-lbs. respectively, − 0·025 per cent. or + 0·03 per cent.

The value of the equivalent obtained from a set of six trials in which the weights had been specially arranged to eliminate the above possible error entirely, gave a result which did not differ at all from that previously obtained, and it may therefore

be safely assumed that in the first series of trials this error did not occur to any sensible extent.

I think that, especially with the above result in view, the loading of the brake may be taken as absolutely accurate.

As to the limit of accuracy of the weighings in the 600 ft.-lb. trials, the weight of water dealt with was approximately 470 lbs. On this quantity the maximum probable error was 0·02 lb. in any trial. This gives greater accuracy than 1 part in 20,000.

The Adjustments of the Brake.

(1.) *Length of the Lever.*—(Part I., par. 45.)

28. This length was required between the centre line of the engine shaft traversing the brake and the V-groove carried by the lever.

It had been previously observed that both the shaft and the brake shifted a little horizontally when the engine was started, from the positions occupied with the engine stationary. It was therefore necessary to make the comparison between the length of the lever and our standard 4-feet with the engine running. Also, since the length of the lever varied with the temperature of the brake, this temperature was maintained, as in all the trials, at 212° Fahr.

Between the brake and the adjacent bearing the shaft is 4 inches diameter within $\frac{1}{1000}$ of an inch.

At a distance of 3 feet 10 inches from one of its square ends a fine line was scribed on a steel straight edge. This straight edge was then held with the square end aforesaid butting against the shaft, the length being horizontal and perpendicular to the line of shafting, and the distance between the straight edge and the lever being 10 inches. At a distance of 11 feet from the other side of the lever a theodolite was set up and adjusted so that the vertical plane of collimation of the instrument was parallel with the shaft and contained the line scribed on the face of the straight edge.

A steel scale, graduated to $\frac{1}{50}$ of an inch, was fixed firmly on to the lever, and a reading of this scale was taken through the telescope without altering the adjustments mentioned. This reading, of course, referred to the point on the scale just 4 feet distant from the centre line of the shaft. By a slight rotation about the vertical axis the line of collimation was then made to cut the centre line of the groove, and then a vertical rotation enabled a second reading of the scale to be taken.

A number of these observations were made while the brake was subjected to moments of 1200, 600, and 400 ft.-lbs., and they all indicated that the length of the lever in the trials made was $4' + 0·02''$.

A correction to the value of the equivalent derived directly from the trials is therefore necessary on this account. It amounts to + 0·0417 per cent.

With this correction added, I think that the length of the lever can be assumed accurate to $\frac{1}{200}$ inch, or 1 part in 10,000 nearly.

(2.) *The Balance of the Brake.*—(Part I., par. 9.)

29. If a pair of trials are run, the one with a heavy indicated load, M_1, and the other with a lighter one, M_2, and if m be the moment carried by the brake on account of its initial want of balance, then the works done in the two trials are

$$U_1 = 2\pi N_1 (M_1 + m)$$
$$U_2 = 2\pi N_2 (M_2 + m)$$

where N_1 and N_2 are the revolutions in the two cases.

The difference of the work done

$$= 2\pi \{N_1 M_1 - N_2 M_2 + m (N_1 - N_2)\}$$

and the relative error involved in writing for this

$$2\pi (N_1 M_1 - N_2 M_2),$$

which has been done in these experiments, is

$$\frac{m (N_1 - N_2)}{N_1 M_1 - N_2 M_2}, \text{ very nearly.}$$

This error is 0 when $N_1 = N_2$.

The speed of the engine was therefore always regulated to the end that the number of revolutions in each of a pair of trials which were afterwards to be compared together should be approximately the same. As a general rule, this object was very nearly attained.

The maximum value of $N_1 - N_2$ was about 300, the values of N_1 and N_2 being approximately 18,000.

Under these circumstances, in trials carrying loads of 1200 and 600 ft.-lbs. respectively, the above error amounts to

$$\frac{300}{18000 \times 600} = \frac{1}{36000} < 0{\cdot}003 \text{ per cent. per ft.-lb. of error in the balance of the brake.}$$

The method pursued to determine the want of balance was as follows :—

The lever was freed from all extraneous loads.

The brake and its pipe connections were then all filled with water, so as to be in the same condition as during the progress of a trial.

The lever was then lifted till its end was in its mean position opposite a pointer at a fixed height from the ground. A load was then gradually added to the front side of the brake till the friction of the bearings was overcome, and the lever fell. An observation of the moment required to cause the motion was then made. A series of

twenty of these observations were made for the front and then a second series of twenty for the back of the brake, in which case the load on the back had to lift the lever from its mean position.

On taking the difference of the means of these two series of observations, the friction is eliminated and the resulting moment represents the error of balance of the brake.

Since in the course of a trial the lever oscillates a little from its mean position, the brake will, when in motion, be working against the resistance offered by the linkage connected with the regulating cock. When at rest, however, this resistance will not affect the load at all. In view of this fact, two determinations of the error in balance were made, the first with the brake working free of the linkage, by allowing the small motion to take place in the slack of the pin-joints, the second with the brake working against the resistance of the regulating apparatus. The results obtained were

In the first case, error in balance = 45·5 ft.-lbs.
In the second case, error in balance = 41·73 ,,

A mean of these two quantities would probably be approximately correct, viz., 43·615 ft.-lbs.

The lead balance weight previously mentioned, and weighing 13·97 lbs., was substituted for one of the 25-lb. weights, on the removal from the lever of the brake of a rider weight and a balance weight whose combined moment (par. 40) was calculated at − 44·12 ft.-lbs.

The actual uncompensated error in the balance appears therefore to be practically $\frac{1}{2}$ ft.-lb. This is so small, and the balancing of the brake such a very difficult operation to perform with any approach to accuracy, that any error there may be has been ignored, and the balance assumed perfect in all the calculations.

The end of the lever has always been kept at the level of the pointer indicated before, and by this means all error due to the varying horizontal position of the centre of gravity of the brake has been avoided.

Terminal Corrections to the Apparent Heat Generated.—(Part 1., par. 31.)

30. In order that the work done in any trial should be exactly equivalent to the heat generated in the water used, it was necessary that the total heat contained in the brake itself should be the same at the beginning and end of the trial.

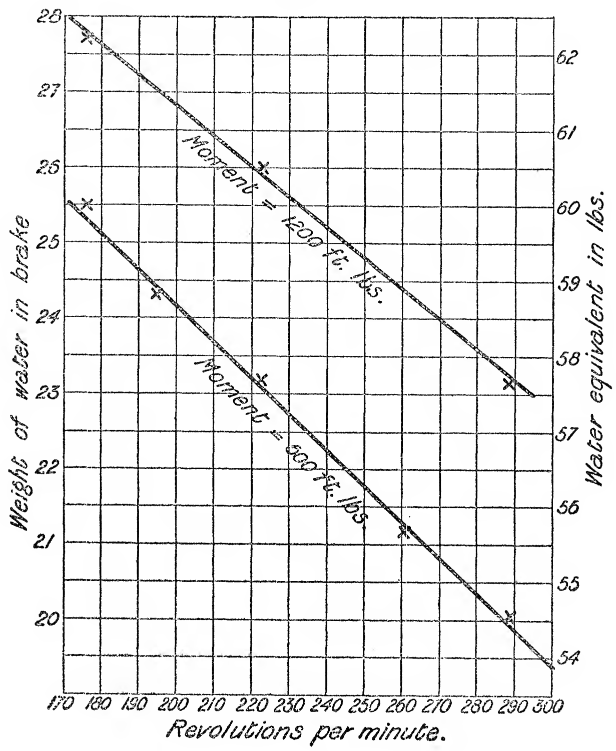
This condition was rarely fulfilled, since it required that the weight of water in the brake, together with its temperature, should be unaltered at the close of the trial.

A determination was made of the amount of water contained by the brake

at various speeds by suddenly stopping the engine when running at any given speed, simultaneously shutting off the water supply to the brake, and afterwards draining off and weighing the water shut in.

The results are shown in the annexed curves.

Fig. 10.



Curves showing water contained by and water equivalents of brake and contents at varying speeds.

The weight of brass in the brake is 368 lbs. Taking 0·094 for its specific heat, the water equivalent is 34·6 lbs.

To obtain a scale of weights representing the water equivalents of the brake at different speeds, we have to add 34·6 to the weights of water contained at the different speeds.

This scale is given at the right of the curves just alluded to (see above).

A correction to the heat obtained is now very easily deduced.

Let w_1 = water equivalent of brake at commencement of trial.
w_2 = ,, ,, end ,,
t_1 = temperature of water in discharge pipe at commencement of trial.
t_2 = ,, ,, ,, end ,,

Therefore, additional heat generated in the brake $= w_2 t_2 - w_1 t_1$, and this quantity is added to the heat already calculated as generated in the water.

The speed indicator which was used in the determination of the number of revo-

lutions per minute required as the ordinate in the curve of water equivalents, was not reliable to one or two revolutions, and, therefore, unless a large difference of speed was indicated between the commencement and end of a trial, this difference was altogether ignored, and the rise in temperature was multiplied by the constant corresponding to any particular load at 300 revolutions to obtain the terminal correction.

The speed gauge required a negative correction of 11 at 300 revolutions, and, consequently, the curves give 57·6 and 54·6 as the water equivalent of the brake when loaded with 1200 and 600 ft.-lbs., respectively.

By interpolation from the above values 53·6 was obtained and used as the water equivalent in trials carrying a moment of 400 ft.-lbs.

Loss of Water by Evaporation and Leakage from the Discharge Pipe and Tank.—(Part I., par 37.)

31. In order to test the general efficiency of the discharge pipe as a conveyer of the water used, it was disconnected in June, 1896, from the brake, and the circulating pump was arranged to pump the water out of the tank and through the discharge pipe, which emptied itself again into the tank by means of the tipping switch.

The stream of water was regulated so as to correspond exactly with the quantities passed in trials carrying loads of 400, 600, and 1200 ft.-lbs. In a period of 62 minutes it was found that in each of these cases the loss approximated very closely to a quarter of a pound of water when its temperature was between 90° and 100°. Since this loss was the same in all the trials it has not been thought necessary to make a correction rectifying the heats on this account, for it would be completely eliminated in the differences of heat used in the calculation of the values of K given in the tables, if the interval of temperature through which the water was raised in the brake was the same in corresponding light and heavy trials.

When, however, I examined the results after the final reduction had been made, I found that the mean temperature of supply in the light trials was 0·7° lower than that in the heavy trials.

Consequently the mean difference of heat would require a slight correction, which, however, is less than − 0·000002 relatively to the whole. This, being quite outside our limits of accuracy, has been ignored.

The Main Experiments.

32. In December, 1895, the apparatus, though not yet quite complete, was in a sufficiently advanced state to make it possible to commence the main K experiments.

The observations were taken and reduced in every experiment in substantially the

same manner that I have described (paras. 17, 18, and 19). Some of the particulars mentioned were, however, omitted in the earlier trials, and were only recorded subsequently after their importance had come to be recognised.

In all, 80 trials were made on which any reliance has been placed, and these will be dealt with in different series, between any consecutive two of which some slight alteration had been made in the apparatus, the method of taking the observations, or of reducing the same; all these alterations leading up to the finally adopted methods which have been described.

33. I must first mention two sets of trials which do not appear in the tables. They were commenced in December, 1895, and were made mainly with the object of gaining experience in the behaviour of the apparatus, and of determining the most favourable conditions under which the experiments could be conducted.

The moments carried by the heavy and light trials in each set were 1200 and 600 ft.-lbs. respectively.

The speed was in the first set 230 revolutions per minute, and in the second set 180 revolutions per minute.

With the following exceptions the apparatus and methods were the same as described.

I. Omissions and faults in apparatus.

(1.) There were no thermometers on either the stuffing-box cover or on the main bearing, and consequently no effectual attempt could be made to keep these parts of the shaft at the same temperature in a pair of trials.

(2.) There was no means of catching the leakage from the stuffing-box, or from the bottom regulating cock.

(3.) The rising pipe at this time only maintained a head of about 5 feet of water over the thermometer in the discharge pipe.

(4.) The hand brake had not been fitted to the shaft.

II. Omissions and faults in the methods employed.

(1.) No corrections were added to the heat as given by the formula $(W_2 - W_1) \times (T_2 - T_1)$.

(2.) The heavy trials were of only half-an-hour's duration, in order that the second reading taken of the weight of the tank should lie on the same part of the scale of the weighing machine, which had not up to this time been corrected, in both heavy and light trials.

The results obtained were not very consistent, but, perhaps largely on that account, the trials admirably fulfilled the purpose for which they were made.

The importance of the terminal corrections was clearly indicated when the results were considered, and consequently means were at once taken to apply these correc-

tions to the preliminary reduction of all subsequent trials. These included the provision of the hand brake, by means of which the engine speed on starting and finishing the trials could be easily controlled, and the observations of the speed of the engine and the temperature of the brake which were taken at the moments of starting and ending the trials.

Again, the terminal corrections and other incidental errors had very unequal weights when acting on the quantities obtained in the hour light trials and in the half-hour heavy trials—which latter quantities required doubling before the subtraction requisite to eliminate losses of heat could be effected.

It was, therefore, decided that in future all trials should be of equal duration (viz. 62 minutes), and this necessitated the immediate careful checking of the scale of the weighing machine, which was thereupon proceeded with. Furthermore, it was probable that many of the discrepancies which occurred were due to the small quantities of water it was possible to deal with at the low speeds hitherto used, and to remedy this defect a larger amount of work was done and heat generated by increasing the speed in all the recorded trials to 300 revolutions per minute. Incidentally this increase of speed was conducive to the steadier running of the engine.

I was much troubled with bubbles of steam in the discharge pipe, and to prevent their formation the rising pipe was lengthened till it gave a head of 11·3 feet over the thermometer bulb.

These trials also furnished information which led to the adoption of a pressure of 9 inches of mercury in the artificial atmosphere. It was found that with higher pressures than this the air by some means found its way into the discharge pipe, even with the lengthened rising pipe in position.

During the first few trials the only regulation of the water supplied to the bearings of the brake consisted of screw clips on the rubber pipes carrying the water. These were found to be very inefficient, and two cocks were substituted, each of which carried a scale which showed the amount to which it was open at any time.

34. Before dealing with the tables showing the final reduction of the experiments made, it is necessary to mention a preliminary reduction of trials Nos. 1 to 42 shown in Table A (p. 413), from which the constants used in the determination of the losses of heat by conduction along the shaft, and also by radiation, were deduced.

In this table the actual observations are as far as possible omitted, since they will appear later in the completely reduced tables.

It will be seen that the table consists of three similar parts, referring respectively to the heavy trials, the light trials, and the differences.

In each part

Col. 1 gives the number of the trial.

Col. 2 gives the work done, calculated in the ordinary way.

Col. 3 gives the heat generated, as calculated from the formula $(W_2-W_1)(T_2-T_1)$, all corrections being omitted.

Col. 4 gives the terminal corrections, for which, as I have said, the necessary observations were always taken.

Cols. 5 and 6 give respectively the mean differences of temperature observed between the stuffing-box and the top and bottom brasses of the main shaft bearing.

The quantities in brackets are not actually observed differences, but were deduced in the manner to be hereafter explained (par. 43).

These differences are + or − according as the stuffing-box was hotter or colder than the adjacent bearing.

Col. 7 gives the mean difference of temperature observed between the brake and the surrounding air. These differences are, of course, all positive.

The quantities given in the part of the table headed "differences" are in every case the remainders which are left on subtracting the corresponding quantities under the heading "light trials" from those appertaining to the "heavy trials."

In the last column are given the values of K, obtained by dividing the work occurring under the heading differences, by the heat, to which has first been added the terminal correction.

The conditions under which each series of trials given in Table A was run are enumerated below.

In every case the engine speed was 300 revolutions per minute, as read on the speed-gauge.

In all heavy trials the moment was 1200 ft.-lbs., with the exception of Series IV., in which the moment was 1244·12 ft.-lbs.

In all the light trials the load was 600 ft.-lbs.

Series I.

35. This series contains trials Nos. 1 to 11, No. 5 being omitted on account of an accident to the revolution counter.

In all these trials the outer brass skin of the brake was exposed directly to the atmosphere, and consequently the loss of heat by radiation was very large.

No attempt was made to catch the small quantities of leakage occurring at the stuffing-box and the bottom regulating cock.

The water supply to the stuffing-box was only regulated to the end that the bearing should not become unduly hot, and no record was kept of the temperature gradient along the shaft till trial No. 10 was reached.

In order to avoid any bias which might be given to the experiments by always combining a trial of one type with one of another type, trials of both of which types were always made at the same relative part of any day, the relative order of running was changed as indicated by the dates and times given in Table B. (Part I., par. 32). This method of combining the trials was adopted because at this time it was not as a rule possible to make more than two trials a day successfully, for breakdowns of a more or less serious nature were of frequent occurrence.

Referring now to the preliminary reduction shown in Table A, Series I.:

The values of K, Nos. I., III., IV., and V. are seen to be in close agreement, notwithstanding the comparatively rough method of reduction used.

Determination No. II., however, stands out as very distinctly higher than the others, and the cause of this was fortunately evident.

In order to prevent the attempted rotation of the small handle shown in the illustrations at the end of the brake lever, one revolution of which altered the load on the brake by 1 ft.-lb., one of my assistants had tied it to the hanger carrying the load. The string making the connection was very tight, and the load was pulled perceptibly out of the perpendicular plane passing through the groove on the lever.

This fault was sufficient to condemn the two trials Nos. 3 and 4, and they do not appear in the final table on that account.

A wooden clip was subsequently added to prevent the rotation of the handle and its attached screw.

Lagging. (Part I., par. 33.)

36. The results given by the four accepted determinations of Series I. were so consistent that it was decided to proceed at once with the lagging of the brake, which, up to the present time, had been deferred on account of want of confidence in the apparatus generally.

The lagging consisted of a layer of about $1\frac{1}{2}$ inches of loose cotton wadding with which the whole of the exterior of the body of the brake was covered, together with the discharge pipe between the brake and the thermometer chamber. The cotton was all tied firmly in position, and the whole was enclosed in a covering of thick flannel.

As will be seen later, this lagging reduced the radiation by nearly 75 per cent. Its weight, about 2 lbs., was inappreciable, and, being evenly distributed, could not affect the balancing of the brake to any extent which it would be possible to detect.

The lagging was, I believe, of use, more especially in that it protected the bare metal from the strong draughts which often occurred in the engine-room. It required very careful attention, however, to protect it against dampness, and on this account I am not certain that better results would not have been obtained without it.

Series II.

37. With the exception of the addition of the lagging, no alteration was made in either apparatus or method between trials 11 and 12.

Sufficient experience and confidence in the apparatus had now been gained to enable me to make three trials per day, as a rule two being made in the morning and one in the afternoon, a stop of about one hour being made after the second trial. The brake was not allowed to cool down during this interval; the hot water contained on finishing the morning's run being shut in.

In Table A, the value 787·4 is given as the result of the combination of trials 12 and 14. There was evidently something amiss with this result, and as the combination of trials Nos. 13 and 14 gave the result 779·4, which agrees fairly closely with those given in Series I., the explanation which at once suggested itself was that the new lagging was damp when the day's running began and had dried before the commencement of trial 13. On this account, trial No. 12 has been expunged from the final Table B, and takes no further part in the investigation.

Series III.

38. As it had by this time been found possible to run three satisfactory trials per day, the most obvious way of combining them was to make three trials, all carrying the same load, on the first day; while the trials required to complete the three determinations were run on the next convenient day.

This method was pursued during the whole of the subsequent course of the investigation.

From this series onward I made an attempt to keep the temperature gradient along the shaft, between the brake and the adjacent bearing, the same in each pair of trials. In trial No. 21, I took observations for the first time of the temperature of the lower brass in the main bearing. In these trials also, the possible importance of the small leakage of water occurring along the spindle of the lower regulating cock, for the first time became apparent. The weight of water actually leaking away had not, I think, any appreciable effect, but owing to its high temperature it was nearly all evaporated, and, consequently, may have had a sensible effect in the lowering of the temperature of the water discharged from the brake. No successful means were yet devised for catching this water. So, in this series, it still remains as a possible source of error.

Series IV.

39. For use in the regular engine trials the brake is provided with a rider weighing 48 lbs., which can be traversed along a graduated scale on the lever by means of a leading screw. In order to maintain the balance of the brake, it carries at the back a second fixed load of 74·6 lbs.

These two large masses of iron had hitherto been left on the brake, but it seemed probable that they would very much affect the flow of heat away from it between any pair of consecutive trials (Part I., par. 33), for they continued to rise in temperature during the whole of any day on which experiments were made, and evidently they would absorb heat more rapidly when cold in the early part of the day than when hot later. It was therefore decided to remove them. Their combined moment about the engine shaft was − 44·12 ft.-lbs.

No allowance was made for this alteration in the loading of the brake, and, consequently, the moment in these trials was 1244·12 ft.-lbs., this figure having been used in the calculations given.

In order to bring the trials under some general denomination, this series has not been further reduced, nor combined with a corresponding set of light trials.

With the intention of stopping the leakage at the bottom cock, I had had some more packing placed in the gland surrounding the cock spindle. This did, to some extent, reduce the leakage, but it also had another effect which will be referred to under Series V.

Series V.

40. For the purpose of keeping the loads on the brake at the values carried by trials preceding the removal of the rider and balance weights, one of the 25 lb. hanger weights was removed, and for it were substituted some lead sheets weighing 13·97 lbs.

This lead weight then corresponded with the initial want of balance to a moment of 100 foot lbs., made up as follows :—

Want of balance	44·12	foot lbs.
Moment of lead weight . .	55·88	,,
	100	,,

After these trials had been made, I determined, with Professor REYNOLDS, by means of a spring balance, the force necessary to move the bottom cock. This was found to amount to a moment of 30 ft.-lbs. on the brake, and on this account this series of trials, though appearing in the final tables, have not been allowed any weight in the calculation of the final mean value of K. The preliminary reduction of Table A gave what were apparently very good values of K, but this only shows the small effect on the mean moment produced by variations in the resistance offered to the brake's motion, and this although its period of oscillation was very long.

Series VI.

41. These trials differ from those of Series V. only in the fact that the extra

packing had been removed from the gland on the cock spindle, while a means of catching the whole of the leakage, and at the same time preventing its evaporation, had been provided (par. 14). The whole of the leakage was credited with the temperature of the water in the discharge pipe, and was weighed with the main stream of water which had been caught in the tank.

Series VII.

42. These trials were made under similar conditions to those in Series VI. In the two last trials, however, viz., Nos. 39 and 42, some leakage was observed and caught from the stuffing-box.

An approximate estimation of the loss of heat due to this leakage is given in Table B, and has been included in the heats given in Table A.

Determination of the Loss of Heat by Conduction along the Shaft.

43. In the trials enumerated in Table A, the varying values of the temperature gradient, existing in the shaft leaving the brake, might evidently be a cause of comparatively large losses of heat which were not eliminated in the differences of heat, so far assumed to be equal to the corresponding differences of work.

It therefore became important to determine, at least approximately, what was the loss of heat by conduction along the shaft in each trial.

I have already said that the temperature of the shaft in the main bearing was assumed to be the same as that of the lower brass, while the temperature on leaving the brake was similarly taken as that of the stuffing-box cover.

Unfortunately, before trial No. 21, I had made no record of the temperature of the lower brass.

It was, however, found that in trials Nos. 21 to 41 the mean temperature of the lower brass exceeded that of the upper brass by about 7° Fahr.

Consequently, in Column 6, in the parts of Table A, where no observations had been taken, an estimation of the difference of temperature between the stuffing-box and the lower brass was made by subtracting seven from the difference occurring in Column 5. In this manner the differences entered in brackets were obtained for trials Nos. 10 to 20.

It appears that we have, therefore, 10 determinations, viz., V., VI., VII., VIII., IX., X., XI., XII., XIII., and XVIII., in which the differences of heat generated require a positive correction on account of the unbalanced conduction along the shaft, and four determinations, viz., Nos. XIV., XV., XVI., and XVII., in which those differences require a negative correction.

Assuming, as is very nearly the case, that the losses of heat by radiation are eliminated in the differences of the heats, it follows that by taking $C =$ loss of heat

per trial, by conduction along the shaft, per unit difference of temperature between the stuffing-box and lower brass,

Then C is given by the equation

$$\frac{675844869}{867995 + 75{\cdot}6\ \mathrm{C}} = \frac{271143956}{348866 - 22{\cdot}5\ \mathrm{C}} = \mathrm{K},$$

where the numerators represent the sums of the differences of work in the sets enumerated above, while the first terms of the denominators represent the sums of the differences of heat in the same sets, to which the terminal corrections have been added. The second term in each denominator represents the correction to be applied to the differences of heat for unbalanced conduction along the shaft.

On solving the equation we get

$$\mathrm{C} = 12, \text{ very nearly.}$$

This agrees very closely with the value $\mathrm{C} = 13{\cdot}61$, which may be calculated from the dimensions of the conducting shaft, viz., 4 inches diameter and $2\frac{3}{4}$ inches long, and FORBES' value of the conduction coefficient for iron, viz.:

$$(0{\cdot}1429 \text{ in C.G.S. unit}).$$

Since nothing was known as to the internal thermal condition of the shaft, the figure 12 has been used throughout as a sufficiently close approximation to the constant required.

The corrections to the heat for conduction along the shaft in each trial were then obtained by multiplying the fall of temperature between the brake and bearing by 12.

The sign of the correction varies, of course, with the sign of the temperature gradient along the shaft.

Determination of the Loss of Heat by Radiation.

44. Under this heading are included all losses of heat not already dealt with under the headings "terminal corrections," "loss by conduction," and "loss by leakage of water."

Radiation in the Unjacketed Trials—Series I.

45. Determination No. II., consisting of a combination of trials 3 and 4, is omitted, for the reasons given. A constant R, representing the loss of heat by radiation per trial per unit difference of temperature between the brake and surrounding air is required.

In Tables B and C, the corrections to the heat are given for terminal errors and conduction along the shaft, the calculation of which has been explained.

The quantities given in the annexed table are sums obtained by adding together the corresponding quantities in Series I. of Tables B and C.

In trials 1, 6, and 9 the loss by conduction has been assumed the same as in trial 10; while in trials 2, 7, and 8 this loss has been given the same value as calculated for trial No. 11.

SERIES I.—Unjacketed Trials.

	Work done.	Heat.	Terminals.	Conduction.	Diff. of temperature between brake and air.
Heavy trials . .	542,876,020	677,309	+ 19	+ 116	556·4
Light trials . .	272,418,189	330,280	− 131	− 496	558·4

We have, therefore, the same value of K given by

$$K = \frac{542{,}876{,}020}{677{,}444 + 556{\cdot}4\,R} = \frac{272{,}418{,}189}{329{,}653 + 558{\cdot}4\,R},$$

and, solving for R, we get

$$R = 36{\cdot}86,$$

or, using this value of R and solving for K,

$$K = 777{\cdot}81,$$

which is the mean value deduced from this series of eight unjacketed trials.

Radiation Coefficient for Jacketed Trials, Nos. 12 *to* 42.

46. As in Series I., we get the sums of work, heat, &c., shown in the annexed table:—

	Work done.	Heat.	Terminals.	Conduction.	Diff. of temperature between brake and air.
Heavy trials . .	1,752,718,746	2,236,681	− 64	− 886	1862·6
Light trials . .	874,319,846	1,108,013	− 183	− 1369	1872·5

In this table the sums are given of the respective quantities in the trials used in Determinations VI. to XVIII. inclusive, Series No. V. being included, because no error was apparent in the quantities obtained; Series No. IV. being omitted, since the moment given could not be guaranteed correct with any certainty.

We thus get the following equation for R :—

$$\frac{874{,}319{,}846}{1{,}106{,}461 + 1872 \cdot 5\ \mathrm{R}} = \frac{1{,}752{,}718{,}746}{2{,}235{,}731 + 1862 \cdot 6\ \mathrm{R}},$$

which, on solution, gives

$$\mathrm{R} = 9 \cdot 33,$$

and, substituting for R,

$$\mathrm{K} = 777 \cdot 91.$$

47. The loss of heat by radiation from the brake, as given in the Tables B, C, &c., was determined by multiplying the difference of temperature between the brake and the air by the radiation constants, calculated as just described.

The Tables B, C, and D, giving the results of trials 1 to 42 inclusive, should now be self-explanatory.

The mean value of K given by the eight unjacketed trials I have mentioned was 777·81.

48. The best way of stating the values of K obtained throughout seemed to be as follows :—

The sums of the differences of the works and of the corrected heats were taken for each series of trials, and then a mean value of K for the series was found by dividing the first of these quantities by the second.

The values of K given as the mean for each series in Table D have been calculated in this way.

49. A mean value of K can be obtained from the jacketed trials contained in Series II., III., VI., and VII. (Series V. being kept out of the determination on account of the possible error already noticed), by finding the sums of the respective differences of work and heat given with each of these series in Table D, and then dividing the work by the heat so obtained.

The sum of the differences of work in Series II., III., VI., and VII.

$$= 676{,}259{,}560,$$

and the sum of the corresponding differences of heat

$$= 869{,}396\,;$$

therefore the mean value of K given by the accepted jacketed trials so far considered is

$$\mathrm{K} = \frac{676{,}259{,}560}{869{,}396} = 777 \cdot 85.$$

From this mean none of the values obtained from any one of the above series differs by as much as 0·03 per cent.

Closer agreement than this could not possibly be expected, and it was consequently

decided to vary the trials somewhat, in order to determine if any errors had been overlooked. For this purpose I made two fresh series of six trials each, the light trials carrying a moment of 400 ft.-lbs. only, none of the other conditions being altered in any way.

50. The full reduction of these Series (Nos. VIII. and IX.) is shown in the two Tables E and F.

As before, three trials were run on each day, but the last trial, on April 1, was not finished on account of an accident preventing me getting the correct weight of the water discharged by the brake. There are, consequently, only eleven trials in the tables. The radiation constant for these trials worked out to 8·16.

The mean value of K, given by the whole eleven trials, was 778·14, which is lower than the two means for the separate series in Table F, on account of the inclusion of the light trial No. 45, which does not appear in Table F.

This new value of K, viz 778·14, did not agree so closely with the former one of 777·85 as we had hoped, and, after reducing the last two series of trials, I devoted all my time to the checking of the whole of the apparatus anew.

It was a consequence of this stringent supervision of every separate part that the small errors in the 25-lb. weights, already noticed, were discovered (par. 27).

51. Calculation showed that this error might account for the discrepancy observed, and so it was decided to run a fresh series of trials with the weights so arranged that no error could appear on their account.

In order to have no known outstanding errors whatever, I made a small rectangular trough, fitted with a drain-pipe, by means of which all leakage from the stuffing-box was caught.

52. A series of fifteen trials, numbered 54 to 68 inclusive, was accordingly made, beginning on June 29, 1896. Owing, no doubt, to the long rest which the apparatus had had since Easter, a number of accidents were met with which completely spoiled the whole series.

The lagging of the brake was very damp when the series was begun, and, on account of the bursting of the various rubber-pipe connections, it did not thoroughly dry during the whole course of this series of trials.

For these reasons the results are not tabulated.

53. After remedying all the defects which had developed in the previous week's running I made two fresh series of six trials each between July 7 and 10 inclusive.

No further accidents occurred and the results were in every way satisfactory.

These are shown in Tables G and H.

The radiation constant worked out at $R = 7·98$.

The mean value of K, given by the two series, was

$$K = 777·85,$$

which happens to be exactly the same as obtained previously from Series II., III., VI., and VII.

54. This last lot of trials afforded no explanation of the small difference (778·14–777·85)

$$= 0{\cdot}3 \text{ ft.-lb. nearly,}$$

which occurred between the results give by the 1200–600 ft.-lbs. determination and the 1200–400 ft.-lbs. determination respectively.

The difference, of course, may be due to terminal errors, which, I think, have been mainly responsible throughout for the small discrepancies found to occur between individual determinations. It is more likely, however, that the small quantity of water dealt with in the 400 ft.-lbs. trials, and the consequent greater effect of the oscillations of the brake on the mean moment, may have introduced some error into these lightly-loaded trials. Further, some slight bias may have been given to the Series, Nos. VIII. and IX., by the long rest caused by the Easter Vacation, between trials 47 and 48.

55. In the annexed table I give the mean value of the work done and of the heat generated in the heavy and light jacketed trials respectively, against which no known sensible error can be placed.

Trials Numbers.	Mean work per trial.	Mean heat per trial.
Heavy trials : (13, 17, 18, 19, 20, 35, 36, 37, 38, 39, 46, 47, 48, 49, 50, 72, 73, 74, 75, 76, and 77)	134,337,403	172,685
Light trials : (14, 15, 16, 21, 22, 23, 33, 34, 40, 41, 42, 43, 44, 45, 51, 52, 53, 69, 70, 71, 78, 79 and 80)	61,355,503	78,867
Differences	72,981,900	93,818

and, dividing the mean difference of work by the mean difference of heat we have

$$K = 777{\cdot}91.$$

This mean value of K deduced from the experiments requires correcting on a few counts, which are due to the method of working. These will be dealt with later.

56. The table given below illustrates the almost perfect manner in which losses of heat were eliminated on the mean result, by the method adopted throughout the investigation of always working on the differences of the quantities of work done and heat generated in a pair of trials.

	No. of revolutions of shaft.	Work done.	Heat generated, less losses due to terminals, conduction, &c.	Loss of heat by leakage of water.	Terminal corrections.	Difference of temperature between stuffing-box and bearing.	Difference of temperature between brake and air.
Means for 21 accepted heavy trials	17,817	134,337,403	171,510	4	− 1	− 3·9	140·5
Means for 23 accepted light trials	17,832	61,355,503	77,710	1	− 7	− 5·4	141·5
Differences	− 15	72,981,900	93,800	3	6	1·5	− 1·0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

A value of K can be obtained by dividing the difference of work in Column 3 by the uncorrected difference of heat in Column 4. This operation gives

$$K = 778{\cdot}06.$$

The various corrections which this number requires are as follows :—

I. Correction due to difference in number of revolutions of shaft between light and heavy trials.

Since the difference in the number of revolutions is only 15, this correction, as previously indicated, when dealing with the balance of the brake, will be zero (par. 29).

II. Correction due to loss by leakage of water from the brake.

This correction amounts to $-\dfrac{3}{93{,}800} = -\ 0{\cdot}000032.$

III. Correction due to terminal differences of temperature of the brake.

This correction amounts to $-\dfrac{6}{93{,}800} = -\ 0{\cdot}000064.$

IV. Correction due to loss of heat by conduction along the shaft.

This correction amounts to $-\dfrac{1.5\ \times 12}{9{,}3800} = -\ 0{\cdot}000192.$

V. Correction due to loss of heat by radiation.

Assuming 9 for the value of the radiation constant, this becomes

$$= +\frac{9}{93{,}800} = +\ 0{\cdot}000096.$$

The total correction factor is therefore (1 − 0·000192), which gives as before

$$K = 777{\cdot}91.$$

Corrections to the Mean Value of K *given by the Experiments.*

I.—Length of Brake Lever.

57. In dealing with the calibration of the measurements of the brake (par. 28), I have already mentioned that the value of K given by the experiments would require a correction factor of $(1 + 0{\cdot}00042)$.

II.—Salts Dissolved in the Manchester Water.

58. Professor DIXON kindly furnished Professor REYNOLDS with the results of a number of analyses of the town's water made during the College session, 1894–95. The dissolved salts were

$$\left.\begin{array}{ll}\text{Common Salt,} & 14{\cdot}4 \\ \text{Calcium Carbonate,} & 27{\cdot}7\end{array}\right\}\text{milligrammes per litre,}$$

therefore the proportion of salts by weight is 0·0000421. Taking their specific heat at 0·2, we get for the correction factor required, due to the lowering of the specific heat of the water,

$$1 + (1 - 0{\cdot}2) \times 0{\cdot}0000421 = (1 + 0{\cdot}00003).$$

III.—Air Dissolved in the Water Used.—(Part I, par. 43.)

59. Being rain water, it probably contained about $2\frac{1}{2}$ per cent. by volume of dissolved air. As affecting the specific heat of the water, this air would not have of itself any sensible influence.

It did, however, influence the resulting final temperature, as it was most probably all boiled out of the water, and the bubbles of expelled air would all be saturated with water vapour at a temperature of 212°, which vapour could not be formed without extracting its latent heat from the surrounding water.

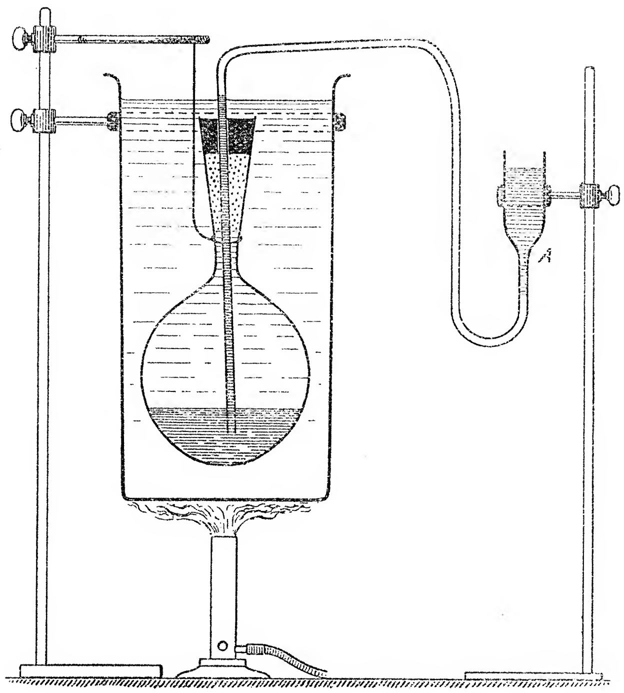
I made some experiments in December, 1896, with the object of determining the actual volume occupied by the bubbles of mixed air and water vapour under the conditions obtaining in the trials. The pressure on the water in the discharge-pipe was 10 inches of mercury very nearly.

The method adopted was as follows :—

I put a depth of about two inches of mercury into the bottom of a strong bolt-head flask, and above the mercury I poured in $1\frac{1}{4}$ lbs. of water. This filled the flask nearly to the brim. A rubber stopper, through which passed a glass tube, was then pressed into the neck of the flask, the glass tube being of such a length that the insertion of the stopper displaced mercury only up the tube, care being taken that no bubbles of air were included under the stopper. The stopper was then firmly tied into the neck, and the flask was hung inside a large glass beaker, which was then filled with water to a depth which covered the top of the rubber stopper.

One end of a piece of strong rubber tube was then fastened on the glass tube protruding from the flask, while its other end was fixed to the vessel shown at A, which was open to the atmosphere.

Fig. 11.



Mercury was poured into the glass funnel at A, and it was raised till there was a solid column of mercury from the bottom of the flask to the surface in A. The water in the beaker was then heated by a Bunsen flame till it boiled. This boiling was continued during a whole day, the water in the beaker being replenished as required. By adjusting the level of the free surface of the mercury at A, any required pressure could be put on the vapour column which formed over the water in the flask neck and displaced some of the mercury from the bottom. Also, by suddenly raising the pressure, the vapour was compressed and cold mercury flowed down into the flask, condensing the vapour in the neck as it descended. By this means the water in the flask could be made to boil briskly for a few moments now and then, so as to facilitate the escape of the air. At the close of the day the levels of mercury and water were adjusted so as to give the requisite pressure on the vapour column. The length of this column was then measured, and knowing the diameters of the flask neck and tube, it was easy to calculate the volume of vapour.

This was 2·2 cubic inches.

If this be reduced to a temperature of 32° and atmospheric pressure, the proportion of air by volume appears to be 1·6 per cent.

This number is considerably less than the 2·5 per cent. already mentioned, but as

it was determined under conditions which approximated closely to those which held in the main trials, it was used in the calculation of the correction given below.

The weight of water vapour at a temperature of 212° per cubic foot = 0·03797 lb.

Therefore the correction due to the loss of the latent heat necessary to evaporate this weight of water, is, relatively to the 180 thermal units generated per lb. of water discharged by the brake,

$$\frac{4}{5} \times \frac{2 \cdot 2}{1728} \times \frac{0 \cdot 03797 \times 966}{180} = 0 \cdot 00021.$$

The correction factor is therefore (1 − 0·00021).

IV.—Reduction of the Weighings to Vacuo.—(Part 1, par. 41.)

60. Taking the density of water = 62·425,

and of air at 32° Fahr. = 0·08073,

and also assuming 70° Fahr. as the mean temperature of the engine-room during the trials, the correction factor becomes

$$1 - 0 \cdot 08073 \times \frac{493}{531} \times \frac{1}{62 \cdot 425} = 1 - 0 \cdot 00120.$$

In the calculation of this factor it must be borne in mind that the density of the air causes errors of equal magnitude in the measurement of both work and heat on account of the alteration of apparent density of the cast-iron weights used on the brake and on the lever of the weighing machine.

V.—Varying Specific Heat of the Water.—(Part 1, par. 51.)

61. According to REGNAULT the mean specific heat of water between freezing and boiling points is 1·005, assuming the specific heat unity at the lower temperature. If his formula for the specific heat be correct, then a correction factor of (1 − 0·00006) is necessary to make the value of K derived from the trials represent this mean specific heat. This factor is introduced because it was not strictly the whole range of temperature between freezing and boiling points which was dealt with in the trials, for the cold water supplied to the brake had various temperatures ranging from 32·7° to 34·3°. This correction would only just affect the second decimal place, and in consideration of the uncertainty that exists as to the exact value of the specific heat of water at any temperature, I do not propose to use a correction factor on this account.

VI.—Corrections due to the Fall in Pressure between the Supply and Discharge Pipes.

62. From observations taken on October 1st, 1896, I determined the pressure on the thermometer in the supply pipe to be :—

In the 1200 ft.-lb. trials	. . . . .	15 inches of mercury.
,, 600 ,, ,,	. . . . .	11 ,, ,,
,, 400 ,, ,,	. . . . .	9·7 ,, ,,

I have already stated that the pressure on the thermometer in the discharge pipe was 11·3 feet of water in all trials.

From these varying pressures two corrections are obtained as follows :—

(*a*.) ELEVATION of Temperature Readings by the Pressure on the Thermometers.

	1200 ft.-lbs.	600 ft.-lbs.	400 ft.-lbs.
Pressure on thermometer bulb in supply pipe in inches of mercury	15·0	11·0	9·7
Consequent elevation in readings of temperature (0°·0072 per inch)	0°·108	0°·0792	0°·0698
Pressure in discharge pipe in feet of water	11·3	11·3	11·3
Consequent elevation in readings of discharge temperature (0°·0066 per inch of mercury)	0°·066	0°·066	0°·066
Percentage correction to heat obtained	$\frac{0·042}{1·8}$ = 0·0233	$\frac{0·013}{1·8}$ = 0·0072	$\frac{0·004}{1·8}$ = 0·0022

If we now confine our attention to the combination of 1200 and 600 ft.-lb. trials, the relative correction to the difference of heat is

$$\frac{0·000233 - \frac{1}{2} \times 0·000072}{\frac{1}{2}} = 0·000394,$$

i.e., the correction factor to K on this account is

$$(1 - 0·000394).$$

Considering next the 1200–400 ft.-lb. determinations, the relative correction to the difference of heat is

$$\frac{0·000233 - \frac{1}{3} \times 0·000022}{\frac{2}{3}} = 0·000339,$$

which makes the correction factor

$$(1 - 0 \cdot 000339).$$

On the mean value of K deduced from the trials, I propose to make this factor

$$(1 - 0 \cdot 00037).$$

63. (*b.*) GENERATION of Heat in the Water on account of the Loss of available Head between the Supply and Discharge Pipes. (Part I., par. 53.)

	1200 ft.-lbs.	600 ft.-lbs.	400 ft.-lbs.
Head in supply pipe in feet of water	17·0	12·45	10·98
Loss of head before reaching the discharge pipe in feet	5·7	1·15	−0·32
Correction required by the work given in the tables per cent.	$\frac{5 \cdot 7}{1 \cdot 8 \times 777}$ $= 0 \cdot 0041$	$\frac{1 \cdot 15}{1 \cdot 8 \times 777}$ $= 0 \cdot 0008$	$\frac{-0 \cdot 32}{1 \cdot 8 \times 777}$ $= -0 \cdot 0002$

Therefore the correction factors required are—

(α) For the 1200–600 ft.-lb. determinations

$$1 + \frac{0 \cdot 000041 - \frac{1}{2} \times 0 \cdot 000008}{\frac{1}{2}} = (1 + 0 \cdot 000074).$$

(β) For the 1200–400 ft.-lb. determinations

$$1 + \frac{0 \cdot 000041 - \frac{1}{3} \times 0 \cdot 000002}{\frac{2}{3}} = (1 + 0 \cdot 000063).$$

This factor also I propose to give the value

$$(1 + 0 \cdot 00007),$$

when applied to the mean value of K deduced from all the trials.

VII.—Correction due to the manner of Engagement of the Revolution Counter with the Engine Shaft. (Part I., par. 34.)

64. The spindle of the counter carried a wire pin parallel with the axis of revolution, which pin was driven by another carried by, and passing at right angles through, the axis of the spindle making connection with the engine shaft.

The mean chance was therefore that at every engagement of the counter with the shaft one-fourth of a revolution would be lost by the instrument, while on disengaging the counter stopped the instant it was withdrawn.

The work in every trial should therefore be increased to compensate for this loss.

The number of revolutions was approximately 18,000.

The correction factor is therefore

$$1 + \frac{1}{72{,}000} = (1 + 0{\cdot}00001).$$

65. A summary of these corrections is appended.

Cause of correction.	Magnitude and sign.	
	+	−
I. Length of lever	0·00042	
II. Dissolved salts	0·00003	
III. Dissolved air	..	0·00021
IV. Weight of atmosphere	..	0·00120
V. Varying specific heat of water	Neglected.	
VI. (*a*) Effect of pressure on thermometers	..	0·00037
(*b*) Loss of head in the water	0·00007	
VII. Engagement of revolution counter	0·00001	
Totals	0·00053	0·00178

Therefore the final correction factor is

$$(1 - 0{\cdot}00125).$$

66. Applying this correction factor to the value obtained from the experiments, we get for the value of the mean specific heat of water between freezing and boiling-points, expressed in mechanical units, at Manchester,

$$777{\cdot}91\,(1 - 0{\cdot}00125),$$

$$\mathbf{776{\cdot}94.}$$

Appendix.

Although no part of this research, it may be interesting to notice that reduced to the latitude of Greenwich this becomes

$$777 \cdot 07,$$

and reduced to latitude 45° at sea level

$$777 \cdot 53.$$

Expressed in metre-grammes and the centigrade unit of heat this last value becomes

$$426 \cdot 58.$$

The value of g being

$$980 \cdot 63,$$

we have for the mean value of the specific heat of water between 0° and 100° C., expressed in absolute C.G.S. units,

41,832,000 ergs.

Making use of REGNAULT's formula for the specific heat of water at different temperatures, this would give the mechanical equivalent of the heat required to raise 1 lb. of water at 60°·5 Fahr. through 1° Fahr. at Manchester as

773·74 ft.-lbs.,

and taking water at 32° Fahr., this gives

773·07 ft.-lbs.

Similarly expressing the result in absolute C.G.S. units, we have for the mechanical equivalent of the heat necessary to raise 1 gramme of water through 1° C. in latitude 45° and at sea-level

(*a*) From a temperature 15°·8 C. . . . . . 41,660,000 ergs.
(*b*) „ „ 0° C. . . . . . 41,624,000 „

TABLE A.—Table Showing the Preliminary Reduction of Trials, 1 to 42 Inclusive.

K Determination number.	Heavy trials. Moment, 1200 ft.-lbs.							Light trials. Moment, 600 ft.-lbs.							Differences.						Preliminary value of K obtained.
	Trial number.	Work done.	Heat generated.	Terminal correction.	Difference of temperature between stuffing-box and upper brass.	Difference of temperature between stuffing-box and lower brass.	Difference of temperature between brake and air.	Trial number.	Work done.	Heat generated.	Terminal correction.	Difference of temperature between stuffing-box and upper brass.	Difference of temperature between stuffing-box and lower brass.	Difference of temperature between brake and air.	Work done.	Heat generated.	Terminal correction.	Difference of temperature between stuffing-box and upper brass.	Difference of temperature between stuffing-box and lower brass.	Difference of temperature between brake and air.	
		Series Number I.																			
I.	1	134,201,612	167,191	+ 11	..	..	139·3	2	68,310,950	82,626	..	..	..	137·4	65,890,662	84,565	+ 11	..	..	..	779·1
II.	4	138,446,542	172,957	− 63	..	..	140·5	3	68,182,773	83,090	− 6	..	..	140·7	70,263,769	89,867	− 57	..	..	..	782·4
III.	6	135,935,775	169,686	+ 31	..	..	137·6	7	67,926,419	82,432	−91	..	..	140·8	68,009,356	87,254	+122	..	..	..	778·3
IV.	9	136,063,953	169,859	− 10	..	..	138·9	8	68,096,065	82,725	−29	..	..	139·1	67,967,888	87,134	+ 19	..	..	..	779·9
V.	10	136,674,680	170,573	− 13	+ 9·4	(+ 2·4)	140·6	11	68,084,755	82,497	−11	− 3·3	(−10·3)	141·1	68,589,925	88,076	− 2	..	(+12·7)	..	778·8
		Series Number II.																			
	12	133,628,584	169,519	− 40	+10·1	..	144·4		Combined	with trial 14.		..	..	..	..	..	..	..	..	..	787·4
VI.	13	135,392,907	172,591	+ 12	+ 9·3	(+ 2·3)	143·8	14	67,933,958	86,054	− 5	−11·2	(−18·2)	142	67,458,949	86,537	+ 17	..	(+20·5)	..	779·4
VII.	17	135,098,853	172,408	+ 6	− 3·9	(−10·9)	140·2	16	67,677,604	85,819	+55	−10·9	(−17·9)	140	67,421,249	86,589	− 49	..	(+ 7)	..	779·1
								15	67,658,754	85,737	−22	− 1·9	(− 8·9)	140·8							
		Series Number III.																			
VIII.	18	133,734,142	170,604	− 69	+ 6·3	(− 0·7)	141·0	21	66,580,557	84,173	− 5	+ 3	− 2·6	145·3	67,153,585	86,431	− 64	..	(+ 1·9)	..	777·5
IX.	19	133,892,479	170,867	+ 63	+ 1·1	(− 5·9)	140·9	22	67,142,275	85,012	−38	− 1·9	− 7·9	144·3	66,750,204	85,855	+101	..	(+ 2)	..	776·6
X.	20	135,332,588	172,666	− 29	+ 0·3	(− 6·7)	140·8	23	66,765,283	84,703	−55	− 2	− 7·7	144·4	68,567,305	87,963	+ 26	..	(+ 1)	..	779·3
		Series Number IV.																			
	24	139,870,565	178,183	+104	+ 2·1	− 6	141·7														
	25	139,448,444	177,847	+ 40	− 1·4	− 7·4	139·1														
	26	140,073,809	178,984	− 40	− 3·1	−10·2	139·3														
		Series Number V.																			
XI.	30	134,073,435	171,054	− 12	+ 7·9	− 1·3	145·8	27	67,353,391	85,344	− 5	− 2·6	−11·1	149·4	66,720,044	85,710	− 7	..	+ 9·8	..	778·5
XII.	31	134,623,843	171,793	− 12	− 6·3	−11·7	145·4	28	67,146,045	85,147	+ 5	−11·8	−18·3	144·2	67,477,798	86,646	− 17	..	+ 6·6	..	778·9
XIII.	32	135,257,190	172,618	..	− 4·0	−10·3	141·4	29	67,315,692	85,406	−16	− 6·2	−13·7	140·4	67,941,498	87,212	+ 16	..	+ 3·4	..	778·9
		Series Number VI.																			
XIV.	35	134,744,481	171,995	+ 6	− 2·9	− 9	144·9	33	67,692,684	85,724	−55	+ 2·1	− 5·1	145·7	67,051,797	86.271	+ 61	..	− 3·9	..	776·7
XV.	36	135,702,040	173,226	− 6	− 3·1	− 9	143·4	34	66,765,283	84,625	..	+ 0·9	− 5·0	144·2	68,936,757	88,601	− 6	..	− 4·0	..	778·1
		Series Number VII.																			
XVI.	37	134,819,879	172,059	− 6	+ 8·9	− 0·3	145·9	40	67,703,993	85,555	−27	+24·9	+11·6	146·5	67,115,886	86,504	+ 21·0	..	−11·9	..	775·7
XVII.	38	135,151,632	172,550	− 17	+ 1·1	− 4·4	144·3	41	67,112,116	85,135	−16	+ 4·0	− 1·7	143·0	68,039,516	87,415	− 0·9	..	− 2·7	..	778·4
XVIII.	39	134,895,277	172,250	..	− 0·3	− 5·9	144·8	42	67,130,965	85,316	−21	..	−16·6	143·1	67,764,312	86,934	+ 21·0	..	+10·7	..	779·3

TABLE B.

Date.	Trial No.	Time of start.	Moment (ft.-lbs.).	No. of revolutions of engine-shaft.	Work done (ft.-lbs.).	Weight of water discharged by the brake (lbs.).	Rise of temperature in the brake (° F.).	Heat generated, less losses by radiation, &c. (B.T.U.).	Weight of water caught at stuffing-box (lbs.).	Rise of temperature in the brake (° F.).	Loss of heat by leakage (B.T.U.).	Rise of temperature of brake during trial (° F.).	Terminal correction to heat (B.T.U.).	Fall of temperature along shaft between stuffing-box and bearing (° F.).	Loss of heat by conduction (B.T.U.)	Difference of temperature between brake and air (° F.).	Loss of heat by radiation (B.T.U.).	Corrected heat (B.T.U.).
Series No. I.																		
Feb. 5, '96	1	11.5	1200	17,799	134,201,612	935·53	178·713	167,191	..	..	..	0·2	11	(2·4)	29	139·3	5135	172,366
Feb. 12, '96	6	1.57	1200	18,029	135,935,775	948·39	178·92	169,686	..	..	..	−0·2	31	(2·4)	29	137·6	5072	174,818
Feb. 13, '96	9	2.8	1200	18,046	136,063,953	952·09	178·406	169,859	..	..	..	0·2	−10	(2·4)	29	138·9	5120	174,998
Feb. 19, '96	10	11.21	1200	18,127	136,674,680	954·46	178·711	170,573	..	..	..	−0·6	−13	(2·4)	29	140·6	5183	175,772
Series No. II.																		
Feb. 28, '96	13	2.14	1200	17,957	135,392,907	965·37	178·782	172,591	..	..	..	0·2	12	(2·3)	28	143·8	1342	173,973
Mar. 5, '96	17	2.52	1200	17,918	135,098,853	964·71	178·715	172,408	..	..	..	0·1	6	(−10·9)	−131	140·2	1308	173,591
Series No, III.																		
Mar. 6, '96	18	10.22	1200	17,737	133,734,142	952·88	179·04	170,604	..	..	..	−1·2	−69	(−0·7)	− 8	141·0	1316	171,843
Mar. 6, '96	19	11.53	1200	17,758	133,892,479	955·85	178·759	170,867	..	..	..	1·1	63	(−5·9)	−71	140·9	1315	172,174
Mar. 6, '96	20	2.26	1200	17,949	135,332,588	969·38	178·12	172,666	..	..	..	−0·5	−29	(−6·7)	−80	140·8	1314	173,871
Series No. V.																		
Mar. 20, '96	30	10.13	1200	17,782	134,073,435	959·88	178·204	171,054	..	..	..	−0·2	−12	− 1·3	− 16	145·8	1360	172,386
Mar. 20, '96	31	11.39	1200	17,855	134,623,843	959·29	179·083	171,793	..	..	..	−0·2	−12	−11·7	−140	145·4	1357	172,998
Mar. 20, '96	32	2.15	1200	17,939	135,257,190	968·3	178·269	172,618	..	..	..	..	..	−10·3	−124	141·4	1319	173,813
Series No. VI.																		
Mar. 25, '96	35	11.1	1200	17,871	134,744,481	962·18	178·756	171,995	..	..	..	0·1	6	− 9·0	−108	144·9	1352	173,245
Mar. 25, '96	36	2.7	1200	17,998	135,702,040	967·85	178·98	173,226	..	..	..	−0·1	− 6	− 9·0	−108	143·4	1338	174,450
Series No. VII.																		
Mar. 27, '96	37	10.27	1200	17,881	134,819,879	963·21	178·631	172,059	..	..	..	−0·1	− 6	− 0·3	− 4	145·9	1361	173,410
Mar. 27, '96	38	11.44	1200	17,925	135,151,632	965·05	178·799	172,550	..	..	..	−0·3	−17	− 4·4	−53	144·3	1346	173,826
Mar. 27, '96	39	2.28	1200	17,891	134,895,277	961·57	179·072	172,190	..	..	60	..	..	− 5·9	−71	144·8	1351	173,530

TABLE C.

Date.	Trial No.	Time of start.	Moment (ft.-lbs.).	No. of revolutions of engine-shaft.	Work done (ft.-lbs.).	Weight of water discharged by the brake (lbs.).	Rise of temperature in the brake (° F.).	Heat generated, less losses by radiation, &c. (B.T.U.).	Weight of water caught at stuffing-box (lbs.).	Rise of temperature in the brake (° F.).	Loss of heat by leakage (B.T.U.).	Rise of temperature of brake during trial (° F.).	Terminal correction to heat (B.T.U.).	Fall of temperature along shaft between stuffing-box and bearing (° F.).	Loss of heat by conduction (B.T.U.).	Difference of temperature between brake and air (° F.).	Loss of heat by radiation (B.T.U.).	Corrected heat (B.T.U.).
Series I.																		
Feb. 5, '96	2	3.4	600	18,120	68,310,950	462·15	178·787	82,626	..	..	..	..	..	(−10·3)	−124	137·4	5065	87,567
Feb. 13, '96	7	10.12	600	18,018	67,926,419	459·03	179·578	82,432	..	..	..	−0·9	−91	(−10·3)	−124	140·8	5190	87,407
Feb. 13, '96	8	11.30	600	18,063	68,096,065	461·58	179·221	82,725	..	..	..	−0·5	−29	(−10·3)	−124	139·1	5127	87,699
Feb. 19, '96	11	2.30	600	18,060	68,084,755	459·66	179·475	82,497	..	..	..	−0·2	−11	(−10·3)	−124	141·1	5201	87,563
Series II.																		
Feb. 28, '96	14	3.36	600	18,020	67,933,958	480·87	178·954	86,054	..	..	..	−0·1	− 5	(−18·2)	−218	142	1325	87,156
Mar. 5, '96	15	10.34	600	17,947	67,658,754	479·29	178·883	85,737	..	..	..	−0·4	−22	(− 8·9)	−107	140·8	1314	86,922
Mar. 5, '96	16	11.50	600	17,952	67,677,604	479·3	179·05	85,819	..	..	..	1·0	55	(−17·9)	−215	140	1306	86,965
Series III.																		
Mar. 12, '96	21	10.33	600	17,661	66,580,557	469·17	179·408	84,173	..	..	..	−0·1	− 5	− 2·6	− 31	145·3	1356	85,493
Mar. 12, '96	22	11.48	600	17,810	67,142,275	474·84	179·034	85,012	..	..	..	−0·7	−38	− 7·9	− 95	144·3	1346	86,225
Mar. 12, '96	23	2.38	600	17,710	66,765,283	473·33	178·951	84,703	..	..	..	−1·0	−55	− 7·7	− 92	144·4	1347	85,903
Series V.																		
Mar. 19, '96	27	10.22	600	17,866	67,353,391	476·05	179·275	85,344	..	..	..	−0·1	− 5	−11·1	−133	149·4	1394	86,600
Mar. 19, '96	28	11.36	600	17,811	67,146,045	474·95	179·276	85,147	..	..	..	0·1	5	−18·3	−220	144·2	1345	86,277
Mar. 19, '96	29	2.19	600	17,856	67,315,692	477·32	178·928	85,406	..	..	..	−0·3	−16	−13·7	−164	140·4	1310	86,536
Series VI.																		
Mar. 23, '96	33	11.38	600	17,956	67,692,684	478·3	179·226	85,724	..	..	..	−1	−55	− 5·1	− 61	145·7	1359	86,967
Mar. 23, '96	34	2.27	600	17,710	66,765,283	472·18	179·221	84,625	..	..	..	..	..	− 5	− 60	144·2	1345	85,910
Series VII.																		
Mar. 30, '96	40	10.49	600	17,959	67,703,993	478·23	178·899	85,555	..	..	..	−0·5	−27	11·6	139	146·5	1367	87,034
Mar. 30, '96	41	11.59	600	17,802	67,112,116	474·96	179·246	85,135	..	..	..	−0·3	−16	− 1·7	− 20	143	1334	86,433
Mar. 30, '96	42	3.29	600	17,807	67,130,965	477·11	178·772	85,294	..	..	22	−0·4	−21	−16·6	−199	143·1	1335	86,431

TABLE D.

Determination No.	Trial No.	Work.	Difference of Work.	Heat (corrected.)	Difference of heat.	K.
			Series No. I.			
I.	1	134,201,612	..	172,366		
	2	68,310,950	65,890,662	87,567	84,799	777·02
III.	6	135,935,775	..	174,818		
	7	67,926,419	68,009,356	87,407	87,411	778·04
IV.	9	136,063,953	..	174,998		
	8	68,096,065	67,967,888	87,699	87,299	778·56
V.	10	136,674,680	..	175,772		
	11	68,084,755	68,589,925	87,563	88,209	777·58
			Mean valne = 777·81			
			Series No. II.			
VI.	13	135,392,907	..	173,973		
	14	67,933,958	67,458,949	87,156	86,817	777·02
VII.	17	135,098,853	..	173,591		
	16	67,677,604	67,421,249	86,965	86,626	778·3
			Mean value = 777·66			
			Series No. III.			
VIII.	18	133,734,142	..	171,843		
	21	66,580,557	67,153,585	85,493	86,350	777·69
IX.	19	133,892,479	..	172,174		
	22	67,142,275	66,750,204	86,225	85,949	776·63
X.	20	135,332,588	..	173,871		
	23	66,765,283	68,567,305	85,903	87,968	779·46
			Mean value = 777·94			
			Series No. V.			
XI.	30	134,073,435	..	172,386		
	27	67,353,391	66,720,044	86,600	85,786	777·75
XII.	31	134,623,843	..	172,998		
	28	67,146,045	67,477,798	86,277	86,721	778·1
XIII.	32	135,257,190	..	173,813		
	29	67,315,692	67,941,498	86,536	87,277	778·46
			Mean value = 778·1			
			Series No. VI.			
XIV.	35	134,744,481	..	173,245		
	33	67,692,684	67,051,797	86,967	86,278	777·16
XV.	36	135,702,040	..	174,450		
	34	66,765,283	68,936,757	85,910	88,540	778·59
			Mean value = 777·89			
			Series No. VII.			
XVI.	37	134,819,879	..	173,410		
	40	67,703,993	67,115,886	87,034	86,376	777·02
XVII.	38	135,151,632	..	173,826		
	41	67,112,116	68,039,516	86,433	87,393	778·55
XVIII.	39	134,895,277	..	173,530		
	42	67,130,965	67,764,312	86,431	87,099	778·01
			Mean value = 777·86			

TABLE E.

Date.	Trial No.	Time of start.	Moment (ft.-lbs.).	No. of revolutions of engine-shaft.	Work done (ft.-lbs.).	Weight of water discharged by brake (lbs.).	Rise of temperature in the brake (° F.).	Heat generated, less losses by radiation, &c. (B.T.U.).	Weight of water caught at stuffing-box (lbs.).	Rise of temperature in the brake (° F.).	Loss of heat by leakage (B.T.U.).	Rise of temperature of brake during trial (° F.).	Terminal correction to the heat (B.T.U.).	Fall of temperature along shaft between stuffing-box and bearing (° F.).	Loss of heat by conduction (B.T.U.).	Difference of temperature between brake and air (° F.).	Loss of heat by radiation (B.T.U.).	Corrected heat (B.T.U.).
							Series No. VIII.											
April 1, '96	46	10.23	1200	17,997	135,688,500	967·92	179·102	173,356	..	..	..	−0·5	−29	−3·4	−41	146	1191	174,477
April 1, '96	47	11.39	„	17,990	135,641,722	969·39	178·73	173,259	..	..	..	−0·3	−17	−5·9	−71	143·8	1173	174,344
							Series No. IX.											
April 17, '96	48	10.43	1200	17,735	133,719,062	955·6	178·505	170,579	..	..	..	..	..	−5·3	−64	143·5	1171	171,686
April 17, '96	49	12.00	„	18,033	135,965,935	974·92	178·162	173,694	..	..	..	..	..	−7·0	−84	142·5	1163	174,773
April 17, '96	50	2.17	„	17,601	132,708,724	950·31	178·352	169,490	..	..	..	..	..	−9·6	−115	141·8	1157	170,532
							Series No. VIII.											
March 31, '96	43	10.30	400	17,958	45,133,482	317·85	179·188	56,955	..	..	..	−0·3	−16	−2·9	−35	145·7	1189	58,093
March 31, '96	44	11.45	„	18,005	45,251,606	318·84	179·158	57,123	..	..	..	+0·1	5	−5·6	−67	143·4	1170	58,231
March 31, '96	45	2.30	„	17,704	44,495,109	311·81	179·495	55,968	..	..	..	0·8	43	−7·4	−89	145·4	1186	57,108
							Series No. IX.											
April 20, '96	51	11.12	400	18,009	45,261,660	318·46	178·777	56,933	..	..	..	−0·1	− 5	−6·1	−73	145	1183	58,038
April 20, '96	52	12.20	„	17,819	44,784,136	316·04	178·686	56,472	..	..	..	0·3	16	−9·1	−109	142·1	1160	57,539
April 20, '96	53	2.39	„	17,919	45,035,464	317·84	178·926	56,870	..	..	..	−0·3	−16	−8·1	−97	142·4	1162	57,919

TABLE F.

Determination No.	Trial No.	Work.	Difference of work.	Heat (corrected).	Difference of heat.	K.
			Series No. VIII.			
XIX.	46	135,688,500	..	174,477		
	43	45,133,482	90,555,018	58,093	116,384	778·07
XX.	47	135,641,722	..	174,344		
	44	45,251,606	90,390,116	58,231	116,113	778·47
			Mean value	= 778·27		

Determination No.	Trial No.	Work.	Difference of work.	Heat (corrected).	Difference of heat.	K.
			Series No. IX.			
XXI.	48	133,719,062	..	171,686		
	51	45,261,660	88,457,402	58,038	113,648	778·35
XXII.	49	135,965,935	..	174,773		
	52	44,784,136	91,181,799	57,539	117,234	777·78
XXIII.	50	132,708,724	..	170,532		
	53	45,035,464	87,673,260	57,919	112,613	778·54
			Mean value	= 778·22		

TABLE G.

Date.	Trial No.	Time of start.	Moment (ft.-lbs.).	No. of revolutions of engine-shaft.	Work done (ft.-lbs.).	Weight of water discharged by brake (lbs.).	Rise of temperature in the brake (° F.).	Heat generated less losses by radiation, &c. (B.T.U.).	Weight of water caught at stuffing-box (lbs.).	Rise of temperature in the brake (° F.).	Loss of heat by leakage (B.T.U.).	Rise of temperature of brake during trial (° F.).	Terminal correction to heat (B.T.U.).	Fall of temperature along shaft between stuffing-box and bearing (° F.).	Loss of heat by conduction (B.T.U.).	Difference of temperature between brake and air (° F.).	Loss of heat by radiation (B.T.U.).	Corrected heat (B.T.U.).
								Series No. X.										
July 7, '96	69	11.17	600	17,548	66,154,556	468·88	178·972	83,916	..	..	..	..	..	−0·57	− 7	135·1	1078	84,987
July 7, '96	70	12.24	600	17,807	67,130,965	474·78	179·474	85,211	..	..	..	..	..	−0·86	−10	135·0	1077	86,278
July 7, '96	71	1.41	600	18,095	68,216,702	482·62	179·608	86,682	..	..	..	..	..	0·57	7	135·6	1082	87,757
								Series No. XI.										
July 10, '96	78	10.45	600	17,436	65,732,325	464·59	179·393	83,344	..	..	..	..	..	−0·71	− 9	136·5	1089	84,424
July 10, '96	79	11.55	600	17,602	66,358,132	470·46	179·269	84,339	..	..	..	−0·4	−22	−4·43	−53	134·3	1072	85,336
July 10, '96	80	1.11	600	17,894	67,458,948	477·08	179·597	85,682	..	..	..	0·3	16	1·14	14	135·1	1078	86,790
								Series No. X.										
July 8, '96	72	11.11	1200	17,311	130,522,170	934·58	178·344	166,677	..	..	..	−0·8	−46	−0·14	− 2	137·7	1099	167,728
July 8, '96	73	12.29	1200	17,528	132,158,316	950·07	177·719	168,845	..	..	..	1·1	63	−0·43	− 5	135·0	1077	169,980
July 8, '96	74	1.43	1200	17,737	133,734,142	956·46	178·559	170,785	0·31	82·3	26	0·3	17	1·0	12	135·5	1081	171,921
								Series No. XI.										
July 9, '96	75	10.35	1200	17,529	132,165,855	945·95	178·39	168,748	..	..	..	0·6	35	0·86	10	134·1	1070	169,863
July 9, '96	76	11.50	1200	17,858	134,646,463	969·82	177·47	172,114	..	..	..	−0·6	−35	−1·71	−20	131·2	1047	173,106
July 9, '96	77	1.6	1200	17,954	135,370,287	974·37	177·566	173,015	..	..	..	0·4	23	−0·57	− 7	130·3	1040	174,071

TABLE H.

Determination No.	Trial No.	Work.	Difference of work.	Heat (corrected).	Difference of heat.	K.
			Series No. X.			
XXIV.	72	130,522,170	..	167,728		
	69	66,154,556	64,367,614	84,987	82,741	777·95
XXV.	73	132,158,316	..	169,980		
	70	67,130,965	65,027,351	86,278	83,702	776·89
XXVI.	74	133,734,142	..	171,921		
	71	68,216,702	65,517,440	87,757	84,164	778·44
			Mean value = 777·74.			
XXVII.	75	132,165,855	..	169,863		
	78	65,732,325	66,433,530	84,424	85,439	777·56
XXVIII.	76	134,646,463	..	173,106		
	79	66,358,132	68,288,331	85.336	87,770	778·03
XXIX.	77	135,370,287	..	174,071		
	80	67,458,948	67,911,339	86,790	87,281	778·07
			Mean value = 777·88.			

DESCRIPTION OF THE PLATES.

PLATE 3.

From a photograph in 1888. Is a front view of the triple expansion engines (100 H.-P.) and brakes, as they existed in the engineering laboratory, Owens College, before any modifications for the determination of the equivalent. The engine-shafts are disconnected from each other, and are working on three separate brakes. In the trials the three large pulleys (5 feet in diameter) were removed with the brakes on the high-pressure and intermediate engines, and the engine-shafts coupled by intermedial shafts, the work being all absorbed by the brake on the low-pressure engine—seen, on the right hand of the plate, overhanging the last bearing of the brake-shaft. On this shaft are two heavy 3-feet pulleys, which served as fly-wheels during the trials.

It was the facilities afforded by this brake and its appurtenances (§ 11) that suggested the research and rendered it possible: and, although the manner of admitting the water and air to the brake were necessarily modified in the experiments, the brake remained essentially the same. Part of the trials were made with the brake uncovered, as seen in this plate; and it was after the brake was covered that subsequent photographs were taken.

The vertical pipe supplying the town's water from the service tank to the brake, with the hand-cock and the automatic inlet-cock above, leading through the bowed pipe and flexible indiarubber tube to the inlet passage over the bush of the brake, are seen on the immediate right. Immediately on the left

and a little behind and lower, is another bowed pipe leading from the top of the brake, with a gap in it; this is the air passage leading through the vanes to the centres of the vortex chambers, to secure atmospheric pressure there. The suspended and riding loads on the lever, the dash-pot, the front stop on which the lever rests (not being at work), are also seen. The hand wheel for adjusting the height of the lever when at work, the linkage connecting the automatic inlet and outlet-cocks with each other and with the front stop, together with the outlet-cock, the receptacle for waste, and the drip-can for the water escaping from the front bush, can be traced, though they are obscure in this plate.

Up high on the photograph is seen a shaft with two large pulleys; these are for connecting the separate engine-shafts by belts and ropes (seen), and have no place in the trials. But the bright shaft immediately below, seen as driven by a rope pulley from behind the wall of the engine-room, is the line shaft driven by the separate engine, always running, which afforded most important facilities for the research.

PLATE 4.

From a photograph, 1896. Also shows a front view of the engine-room, but, taken more to the right; it includes only the low-pressure engine. It shows a general front view of the appliances in the condition in which they were during the final experiments, as well as some of the standing appliances not included in Plate 3.

Low down, immediately on the right, is the front of the weighing-machine, with the tank resting on it; and immediately behind this, against the wall, are seen the mercury balances for the pressures of water in the mains; also the town's main to the service tank (out of sight on the right), in front of which is the 3-inch quadruple turbine which drives the ($1\frac{1}{2}$-inch) quintuple centrifugal pump (out of view, behind the tank) supplying the brake through the ice-cooler (§ 20). On the left of the tank, and passing through its cover, is the water-switch; and over this is the nozzle of a vertical pipe, straight almost to the roof, then horizontal, with an open vertical branch, to form an air-gap, then down again into the lower of the two horizontal pipes; this is the stand-pipe on the outlet from the condenser, for securing pressure in the final thermometer chamber (§ 22). The upper of the two horizontal pipes is the water-jacketed out-flow pipe or "condenser," which passes to the end of the room, and returns as the lower horizontal pipe to the stand-pipe. Immediately on the left of the plate, standing on the floor, is the frame for the hand-brake (§ 30). Besides the appliances mentioned, as seen, in this plate, nearly all the appliances are seen in front view; but many are better seen in the following plates, though this plate affords the best view of the general arrangement, and the best idea of the circumstances under which the observations were made. The passage between the brake and the 3-inch pipe supplying condensing water to the engine afforded the only post of observation for the counter, thermometers, speed-gauge, and pressure-gauges. The centrifugal speed-gauge, with its scale, is seen rising vertically from behind the small pressure-gauge on the brake.

PLATE 5.

This is a nearer and simplified front view of the more special appliances shown in Plate 4. Proceeding from the right is the switch and outlet nozzle from the condenser, with the water flowing into the tank over the thermometer. From the switch may be traced the linkage forming the automatic connection of the switch with the counter, immediately in front of the covered bush of the brake. Supported by the original supply pipe to the brake (the hand cock being shut) is seen the new inlet pipe from the ice-cooler, behind the brake. The pipe, rising on the right from behind the brake, passes a branch to the by-channels leading to the bushes (not seen) and a branch to the large pressure-gauge, then to the regulator; thence the water flows upwards past the bulb of the inlet thermometer, some of it passing up through the glass thermometer chamber, and so to waste through the small pipe at the top, but the main stream passing through the covered horizontal branch, and down the flexible indiarubber pipe

into the brake. On the top of the brake is seen the new air-passage, of flexible indiarubber, leading to the vessel in which is the artificial atmosphere, which is connected with the large mercury-gauge on the left, also with the syringe. The automatic outflow cock is clearly seen under the brake, also the curved flexible pipe, covered with cotton wool, which receives the water from the outflow cock, leading to the fixed pipe behind the regulator, also covered, in which is the bulb of the outflow thermometer, and immediately over this the glass thermometer chamber, with its indiarubber continuation leading back into the main outflow channel which rises up behind the inlet thermometer chamber, till it turns at right angles into the condenser. Behind and on the left of the brake are seen protruding the stems of the thermometers for measuring the difference of temperature in the stuffing-box and the near bearing. Of the two bottles standing on the floor, that on the left is collecting the leakage from the stuffing-box, and the other the leakage caught in the indiarubber bag enclosing the automatic outflow cock.

PLATE 6.

This is a back view. On the left, close in front of the tank on the weighing machine, over which is the condenser leading to the switch, is seen the $1\frac{1}{2}$-inch quintuple centrifugal pump, with its driving gear and the pipe supplying it from the service tank. On the other side of the 3-inch pipe for condensing water for the engines, and partly behind it, is seen the pipe leading from the pump up and along behind the 3-inch pipe, then down again into the ice-tank (on the extreme right of the plate); through this it passes in a coil, emerging from the cover again as the covered pipe rising obliquely to the regulator and inlet thermometer chambers (not seen), with the branch to the pressure-gauge. The small horizontal branch coming through from beneath the pressure-gauge, continued by the covered indiarubber pipe, passing behind the vortex vessel of the speed-gauge to the stuffing-box, is one of the by-paths taking ice-cold water to the bushes; that on the left is behind the brake. The outlet thermometer chamber, with its indiarubber continuation to the main outflow channel into the condenser, is also clear; as are also the belt and pulley driving the paddle in the ice-tank.

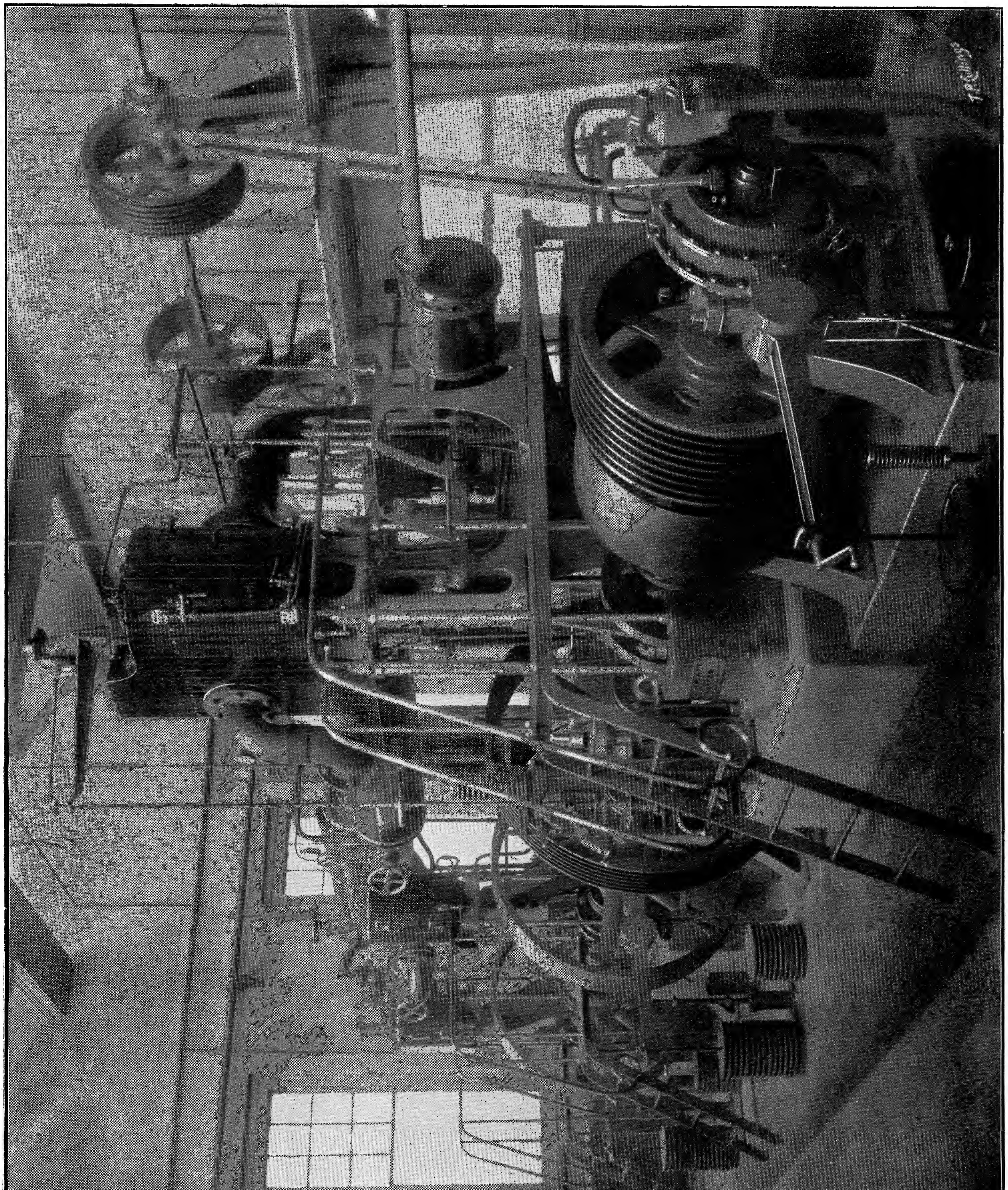
PLATE 7.

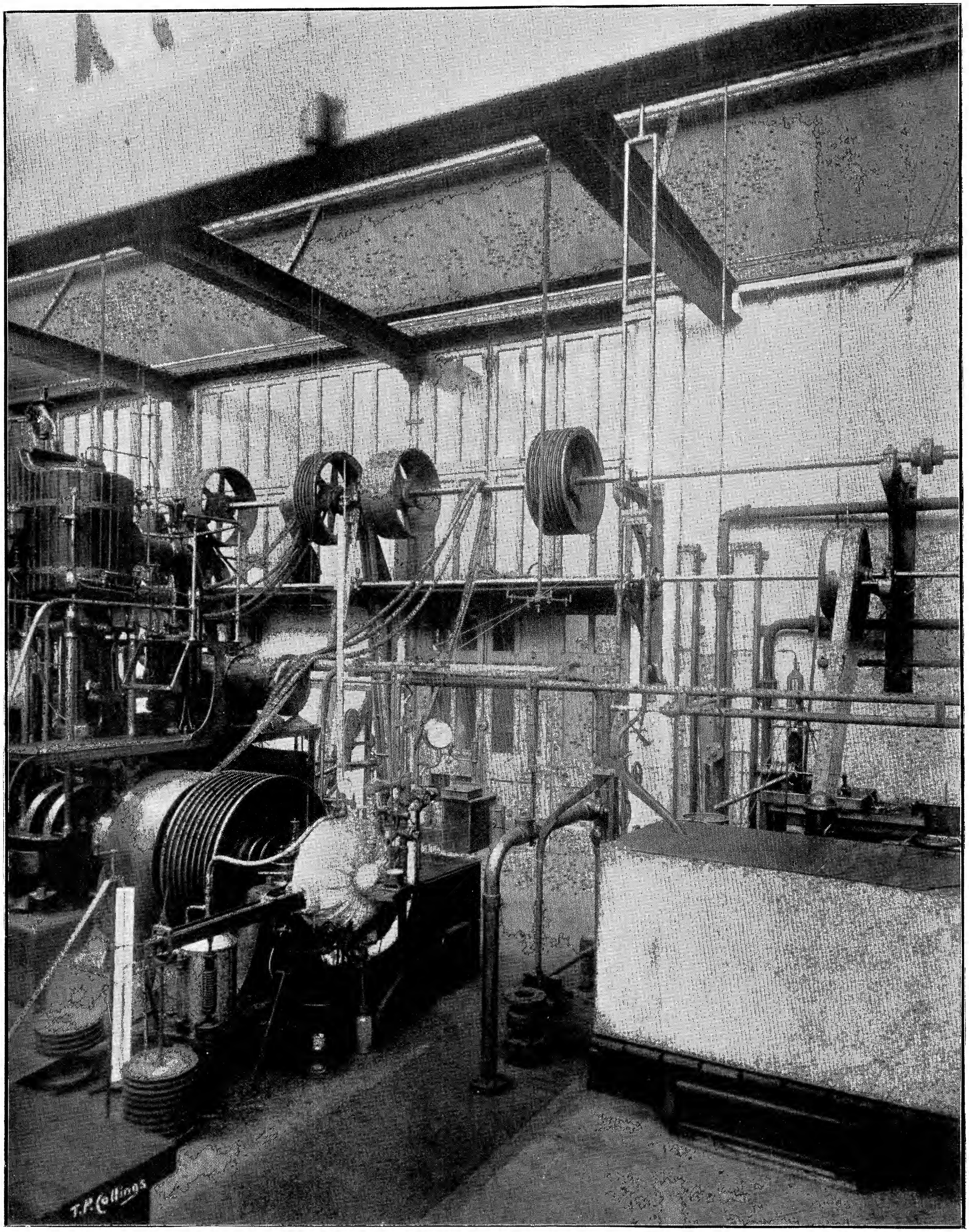
This is again a back view, but taken so as to show the appliances up to the end of the engine-room, not seen in the previous plates. In the middle front is seen the 6-inch quadruple centrifugal pump in circuit, with the rising 4-inch main from the lower tank to the tank in the tower (§ 3), together with the belt from the line shaft by which this pump is driven. Immediately on the left of this plate, standing on a bench, is the end of the 3-inch quadruple vortex turbine, driven by water from the tower, and driving by a cord the $1\frac{1}{2}$-inch quintuple centrifugal pump. The standard, the lever, and the large riding weight of the weighing-machine, with the tank behind, are completely in view; and over these again appears the condenser for cooling the effluent water, passing to the end of the room and returning underneath to the stand-pipe and thence to the switch.

PLATE 8.

This is from a photograph of the apparatus for correcting the high temperature thermometer. On the table is the barometer, and to the right is the vapour chamber, in which the thermometer is immersed through the cork on the top as far as to leave the top of the mercury visible. The escape passage and regulator are seen on the right. The pipe leading from the top is the connection of the vapour chamber with the lower mercury chamber in the barometer. This, after passing through the flask, receives by the branch (seen) a slight current of air from the pressure reservoir, with the top of which it is connected by a restricted pipe, so that the current is so slow that the resistance is negligible, though sufficient to prevent the vapour passing to the barometer; the pressure of air in the reservoir is shown by the large

mercury-gauge, and is maintained by occasional pumping with the syringe seen in connection. The nozzle on the barometer, to which the air-passage is connected, leads into the cast-iron bottle which forms the mercury-chamber, above the surface of the mercury. The level of this surface is observed through the circular windows, of which that which is in front is shown to the left of the axis of the barometer, above the nozzle. Immediately above this window is seen the cylindrical brass curtain, which screws on to the neck of the bottle, by which the light through the windows over the mercury can be eclipsed. Attached to this curtain, and co-axial with it, is the outer brass tube extending up to the gap, with a vertical scale attached reaching past the gap. Behind the vertical scale, and screwed into the tube on the lower curtain, is a tube screwed throughout its length, and having two parallel slots, as windows, some 5 inches long, through which the upper limb of the mercury may be observed. From the top of this windowed tube downward is screwed the cap, the lower limb of which forms a cylindrical curtain for eclipsing the light over the upper limb of the mercury (§ 48).

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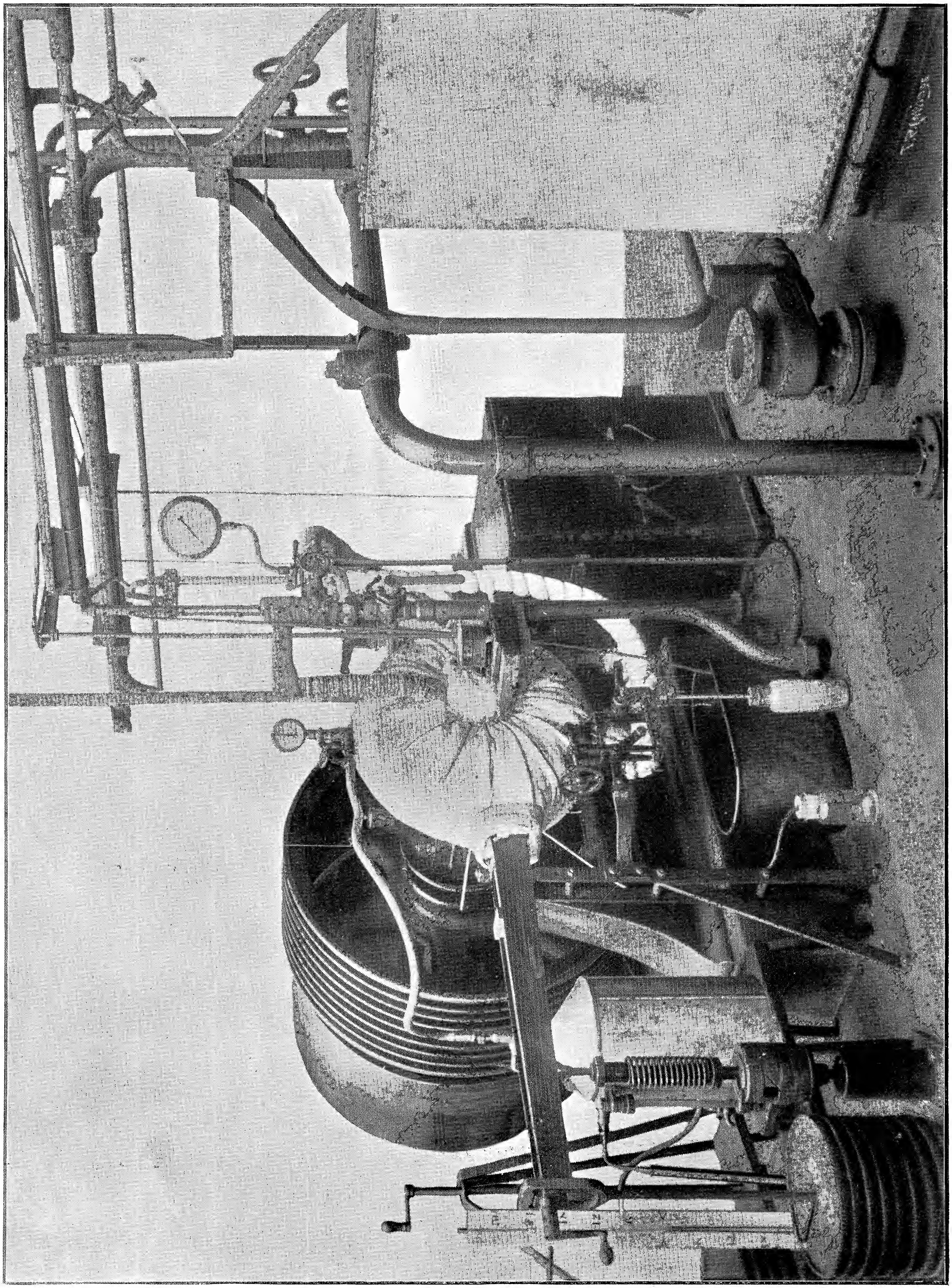

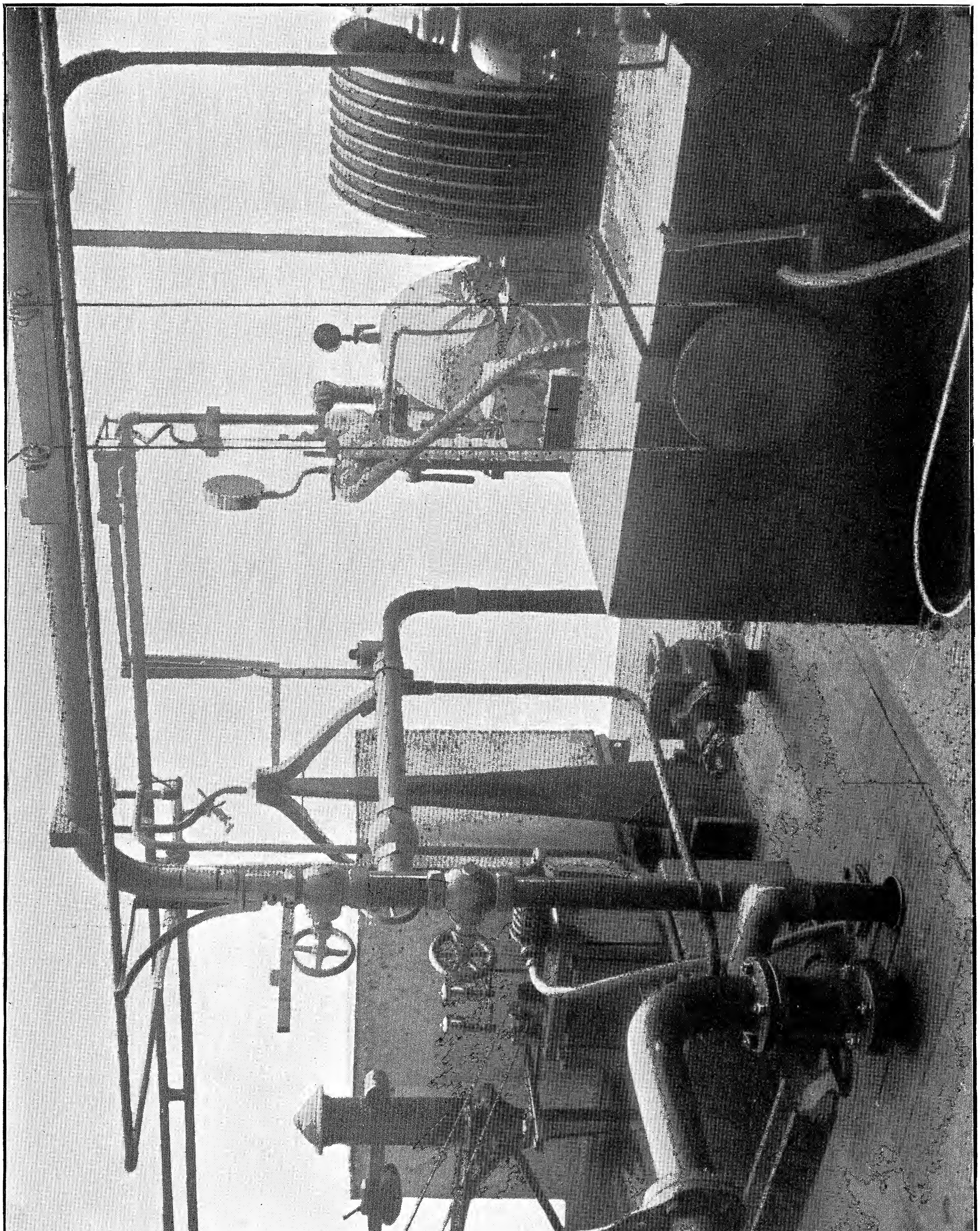

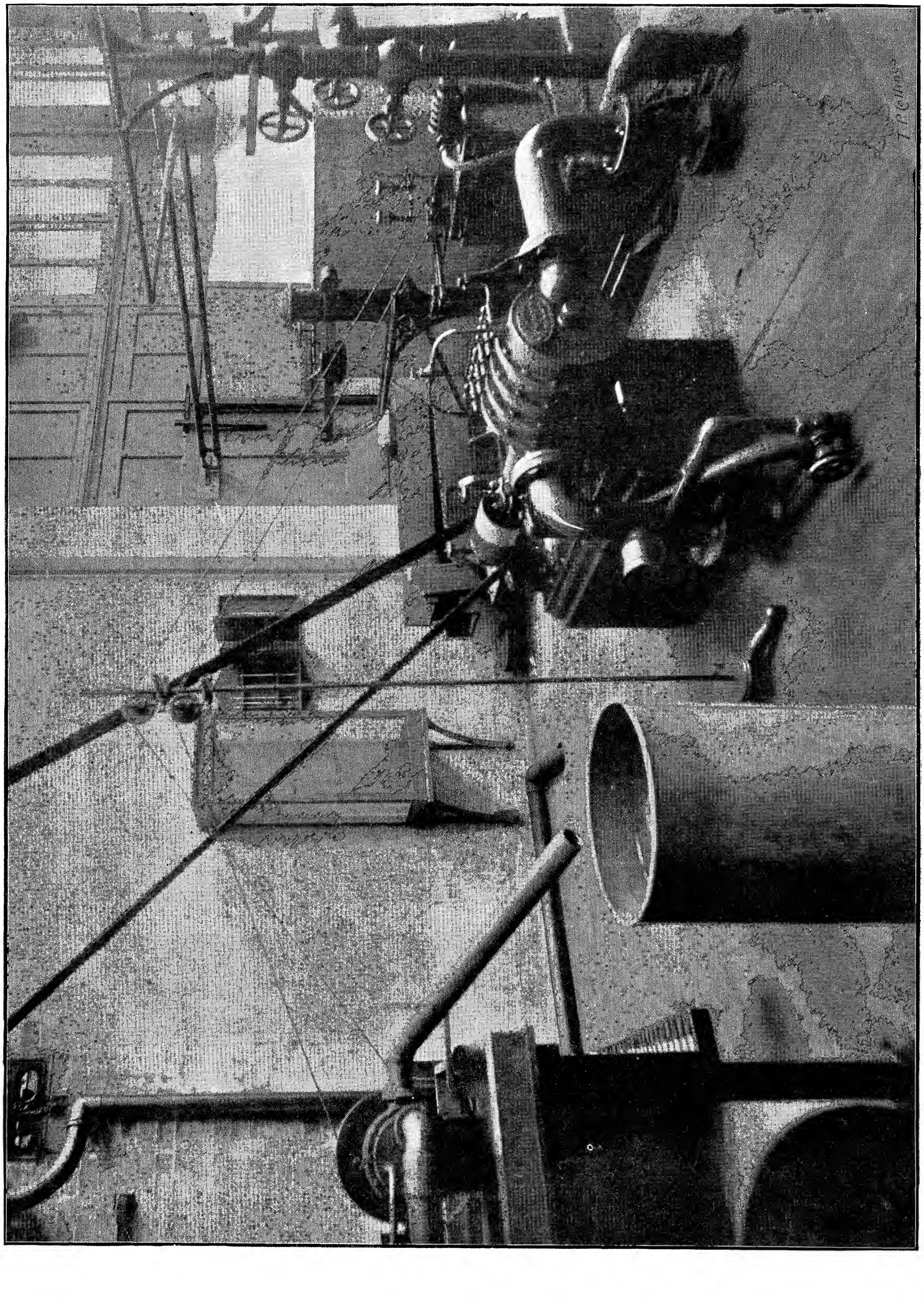

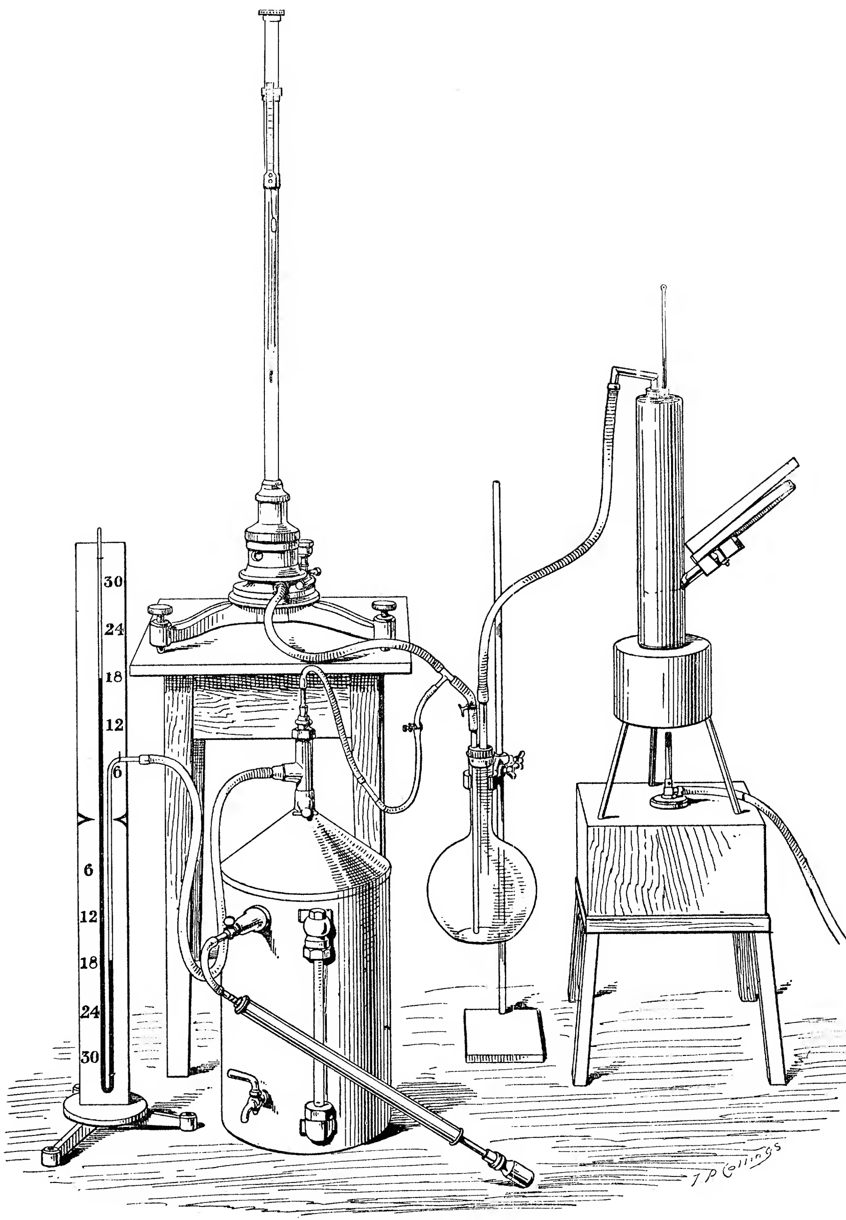
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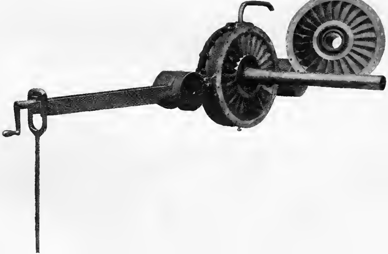
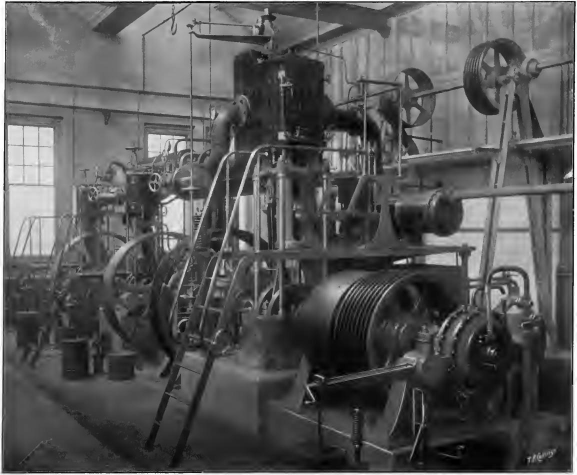
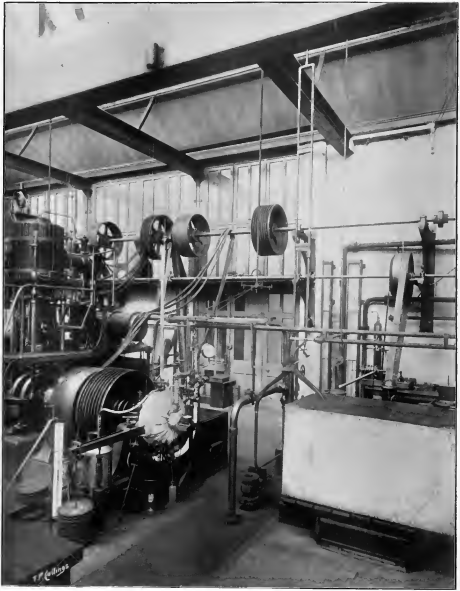
Fig. 2.



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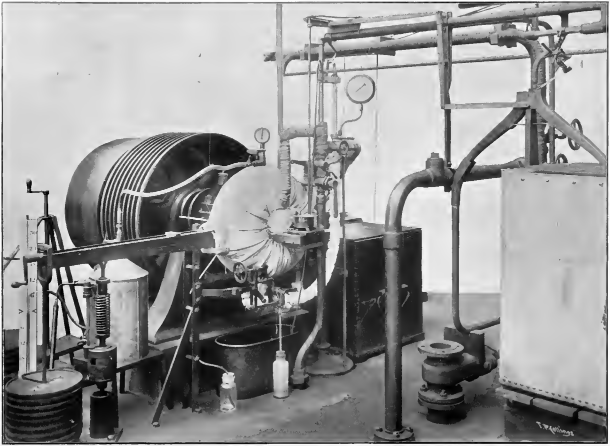

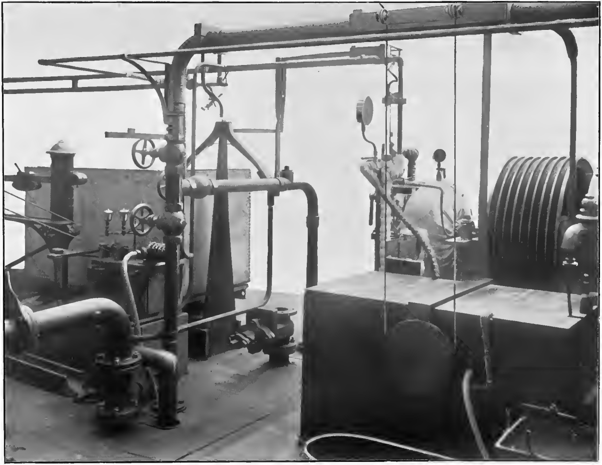

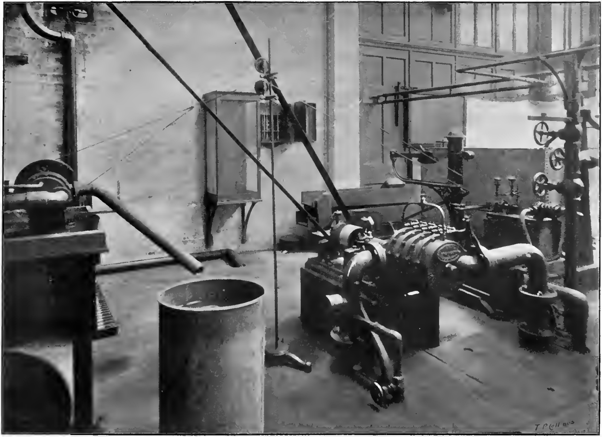
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